École Polytechnique Fédérale de Lausanne
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Discrete Optimization (Spring 2019)

Assignment 4

Problem 1

Consider the polyhedron:

$$P = \begin{cases} x_1 + 2x_2 + x_3 & \leq & 5 \\ 3x_1 + x_2 + x_3 & \leq & 3 \\ x_1 & \leq & 1 \\ x_1 + x_2 & \leq & 2 \\ x_2 + x_3 & \leq & 3 \\ x_1 & \geq & 0 \\ x_1 + x_2 & \geq & 0 \\ x_2 + x_3 & \geq & 0 \end{cases}$$

State which of the following points are vertices of P: $p_0 = (0,0,3)$, $p_1 = (0,1,1), p_2 = (1,4,-4)$, $p_3 = (1/2,3/2,0), p_4 = (1,-1,1)$.

Problem 2

Consider the following classification problem: we are given $p_1,\dots,p_N$ points in $\mathbb{R}^d$, and each point is colored either blue or red. We want to determine if there is an hyperplane $\alpha = \{ax = b\}$ that strictly separates the blue points from the red ones (i.e. such that $ap_i > b$ for all blue points and $ap_i \leq b$ for all red points) and, in case of a positive answer, find such α. Show how to solve this problem using linear programming.

Problem 3

Recall the linear program from the last assignment:

$$\begin{aligned} \max \quad & a + 3b \\ \text{s.t.} \quad & a + b \leq 2 && (1) \\ & a \leq 1 && (2) \\ & -a \leq 0 && (3) \\ & -b \leq 0 && (4) \end{aligned}$$

Solve it with the Simplex method starting with the initial feasible basic solution induced by the constraints (2) and (4). For each iteration indicate the current basis and the corresponding vertex, λ_B, the direction in which the Simplex moves and how far it moves. At the end indicate the optimal objective value and the proof of optimality (i.e. the final λ).

Problem 4

Consider the following linear program:

$$\begin{aligned} \max \quad & 6a + 9b + 2c \\ \text{subject to} \quad & a + 3b + c \leq -4 && (1) \\ & b + c \leq -1 && (2) \\ & 3a + 3b - c \leq 1 && (3) \\ & a \leq 0 && (4) \\ & b \leq 0 && (5) \\ & c \leq 0 && (6) \end{aligned}$$

Solve the linear program with the Simplex method and initial vertex $(-1,-1,0)^T$. For each iteration indicate all the parameters as in the previous exercise including the optimal value and the proof of optimality.

Problem 5

Let $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and A has rank equal to n. The goal is to show that we can find an initial vertex (or show that P is empty) by solving an appropriate LP:

1. Show that P is non-empty (feasible) if $b \geq 0$.
2. Show that $Q = \{(x, y) \in \mathbb{R}^{n+m} \mid Ax \leq b + y,\ y \geq 0\}$ has a vertex v.
3. Create a LP for Q (with initial basic solution corresponding to v) that either shows whether P is feasible or not.