The problem can be submitted until March 8, 12:00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) 1:

**Question 1:** The question is worth 5 points.

 $\square \ 0 \ \square \ 1 \ \square \ 2 \ \square \ 3 \ \square \ 4 \ \square \ 5$  Reserved for the corrector

Assume validity of the following form of the Farkas' lemma

Let  $A \in \mathbb{R}^{m \times n}$  be a matrix and  $b \in \mathbb{R}^m$  be a vector. The system  $Ax = b, x \geq 0$ ,  $x \in \mathbb{R}^n$  has a solution if and only if for all  $\lambda \in \mathbb{R}^m$  with  $\lambda^T A \geq 0$ , one has  $\lambda^T b > 0$ .

Prove the following variant of Farkas' lemma (Theorem 3.11):

Let  $A \in \mathbb{R}^{m \times n}$  be a matrix and  $b \in \mathbb{R}^m$  be a vector. The system  $Ax \leq b, x \in \mathbb{R}^n$  has a solution if and only if for all  $\lambda \in \mathbb{R}^m$  with  $\lambda^T A = 0$  one has  $\lambda^T b \geq 0$ .

<sup>1.</sup> You are allowed to submit your solutions in groups of at most three students.