École Polytechnique Fédérale de Lausanne
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Discrete Optimization (Spring 2019)

Assignment 1

Problem 1

Picture yourself in the role of a production manager. There are n production tasks to be executed, and a task j requires p_j working hours to be completed.

You have m employees at your disposal that can each, due to his or her qualifications, work on a subset of the tasks. Denote by S_i the set of jobs that employee i can work on.

A work allocation plan has to ensure that all tasks are completed. You want to create an allocation which is also fair: the maximum number of working hours assigned to an employee is to be minimized. Model this problem as a linear program.

Problem 2

Consider the problem

$$\begin{array}{rl} \min & 2x + 3|y - 10| \\ \text{subject to} & |x + 2| + y \leq 5, \end{array} \tag{1}$$

and reformulate it as a linear programming problem.

Problem 3

Prove the following statement or give a counterexample: The set of optimal solutions of a linear program is always finite.

Problem 4

Consider the following linear program:

$$\begin{array}{lrcrcl} \max & x & + & y & & \\ \text{s.t.} & 3x & + & 2y & \leq & 6 \\ & x & + & 4y & \leq & 4. \end{array}$$

The solution $(x, y) = (8/5, 3/5)$ satisfies the both constraints and has the objective value $11/5$. Provide a certificate that this is an optimal solution.

Problem 5

Let (2) be a linear program in inequality standard form, i.e.

$$\max\{c^{\mathrm{T}}x \mid Ax \leq b, x \in \mathbb{R}^n\}, \tag{2}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$.

Prove that there is an equivalent linear program (3) of the form [1]

$$\min\{\tilde{c}^{\mathrm{T}}x \mid \tilde{A}x = \tilde{b}, x \geq 0, x \in \mathbb{R}^{\tilde{n}}\}, \tag{3}$$

where $\tilde{A} \in \mathbb{R}^{\tilde{m}\times\tilde{n}}$, $\tilde{b} \in \mathbb{R}^{\tilde{m}}$, and $\tilde{c} \in \mathbb{R}^{\tilde{n}}$ are such that every optimal point of (2) corresponds to an optimal point of (3) and vice versa.

[1] This is called *equality standard form.*