Fall 2014 Randomized Algorithms Deadline: Oct 8, 2014

Problem Set 1

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1. Is the $\binom{n}{2}$ upper bound on the number of minimum cuts in an undirected connected graph tight? Justify your answer. **[3 pts]**

2. Instead of choosing random edges to contract, if we repeatedly choose a pair of vertices uniformly at random to coalesce show that the resulting algorithm could have an exponentially small probability of finding a minimum cut on some inputs. **[3 pts]**

3. Show how to use the randomized edge contraction algorithm to find all the global minimum cuts in an undirected multigraph. **[3 pts]**

4. Consider Seidel's recursive linear programming algorithm for the case of a non-empty, bounded, and non-degenerate polyhedron in d dimensions defined by m constraints. Let $T(d, m)$ denote the expected running time of the algorithm to find an optimum on such an instance. Obtain an upper bound on $T(d, m)$ that is linear in m. **[3 pts]**

5. For any $\alpha \geq 1$, define an α-approximate cut in a multigraph G as any cut whose cardinality is within a multiplicative factor α of the cardinality of a mincut in G. Prove that the number of α-approximate mincuts in a multigraph G is at most $O(n^{2\alpha})$. **[6 pts]**

6. An r-way cut is a collection of edges whose removal partitions the graph into at least r connected components. Prove that the number of minimum cardinality r-way cuts in a connected undirected graph is at most

$$\frac{1}{r}\binom{n}{r-1}\binom{n-1}{r-1}.$$

[6 pts]