Prof. Friedrich Eisenbrand
Dr. Yuri Faenza

Discussions from: 17 December 2013

Combinatorial Optimization

Fall 2013

Assignment Sheet 7

Exercise 1
This immediately follows from the definition of ellipsoid and from the fact that, for $x = b$, we have $||A^{-1}(x-b)|| = 0$, hence by continuity of linear mappings $\{x : ||A^{-1}(x-b)|| \leq 1\}$ is full-dimensional.

Exercise 2
See Lemma 7.1 in Prof. Eisenbrand's lecture notes, http://disopt.epfl.ch/files/content/sites/disopt/files/shared/Opt2011/lecture_115.pdf.

Exercise 3
By definition, a facet F of P is a set of the form $P \cap \{x : cx = \delta\}$. We observed it has dimension $n-1$. If $cx = \delta$ were not a multiple of $ax = \beta$, then $F = P \cap \{x : cx = \delta, \, ax = \beta\}$ is of dimension $n-2$, a contradiction. Hence $cx = \delta$ if and only if $ax = \beta$, and the statement follows.

Exercise 4
Consider an instance of maximum matching over a graph G with weights w. Make a copy of G (keeping the same weight function) and connect each element of G with its copy via an edge of cost 0. Let G' be the graph and w' the cost function obtained. One easily checks that a perfect matching of G' is of maximum weight wrt w' if and only if it induces a maximum weighted matching in G wrt w. Now consider an instance of maximum weight perfect matching (again, over G wrt w). Add to the weight of each edge a value $M >> 0$, as to get the weight function w'. Any maximum matching of G wrt w' will be a perfect matching of G maximum cost (wrt w).

Exercise 5
Let $x \in \mathbb{R}^n$ be the point of P that is given. Let S be the set of inequalities that are satisfied at equality by all points of P. S can be found in polynomial time by, e.g., perturbing the right hand side of one inequality at the time, and checking if the corresponding system is feasible. The intersection of all inequalities from S is the affine hull L of P. Take any direction $v \in L \setminus \{0\}$ and let $\alpha \geq 0$ be the maximum scalar such that $x' = x + \alpha v \in P$ (this can be found in polynomial time, for P has a polynomial number of inequalities). Let S' be set of constraints from P that are satisfied at equality by x', and note that L' (define similarly to L above, just

starting from S') has bigger dimension than L. Hence, after at most n repetitions, we end in a point of P that satisfy n linearly independent constraints from P, that is, in a vertex of P.

Exercise 6
See pages 215-218 of the book Combinatorial Optimization by William J. Cook, William H. Cunningham, William R. Pulleyblank, A. Schriver, Eds. Wiley-Interscience.