Exercises

Optimization Methods in Finance

Fall 2009

Sheet 5

Exercise 5.1
Let $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ be a polyhedron with $P \neq \emptyset$ and let $\varepsilon > 0$. Show that

$$P_\varepsilon = \{x \in \mathbb{R}^n \mid Ax \leq b + \varepsilon \cdot \mathbf{1}\}$$

is fulldimensional (here $\mathbf{1} = (1, \ldots, 1)$).
Hint: Recall that P_ε is fulldimensional if there exists a $y \in P_\varepsilon$ and a $\delta > 0$, such that the ball of radius δ around y is contained in P_ε, i.e. $\{x \in \mathbb{R}^n \mid \|x - y\| \leq \delta\} \subseteq P_\varepsilon$.

Exercise 5.2
Suppose we have a fixed budget of $B \in \mathbb{N}$ and we want to invest this money into bonds $i = 1, \ldots, n$. Buying once bond i costs $w_i \in \mathbb{N}$ today and gives us a profit of $p_i \in \mathbb{Q}_+$ next year. Suppose that we can buy an arbitrary (non-negative) integer amount of every bond. We want to make a decision that maximizes the cumulated profit next year (not invested money does not give a profit) using dynamic programming.

i) Define suitable states/table entries and a Bellman equation that will give you an optimum solution.

ii) Compute the concrete table entries for $B = 7$ and the following setting

bond	w_i	p_i
$i = 1$	5	2.6
$i = 2$	2	1.0
$i = 3$	3	1.9

iii) What is (roughly) the running time of your dynamic programming algorithm (for general n)? What would be the running time of a naive algorithm that tries out all combinations? Compare them for say $n = B = 100$ if your computer can process 1 billion operations per second.

Exercise 5.3
Suppose we are given a map of canton Vaud with n many villages. For villages i and j, let $c_{ij} \geq 0$ be the length of the road from i to j (or $c_{ij} = \infty$ if no direct road exists). Consider table entries

$A(i, j, k) =$ length of the shortest path from i to j, where we use at most k many intermediate stations

Give Bellman equations to compute $A(i, j, k)$. How is the base case $A(i, j, 0)$ defined? How can we read the length of the shortest route (using arbitrarily many interstations) from i to j from the table?

Exercise 5.4

Compute the value of an American call option on a stock with current price equal to 100 CHF, strike price equal to 102 CHF and expiration date four weeks from now. The yearly volatility of the logarithm of the stock return is $\sigma = 0.30$. The risk-free interest rate is 4%. Use a binomial lattice with $N = 4$.