Integer Points in Polyhedra

Spring 2009

Assignment Sheet 11

Exercise 1 (Integer programming gaps)
Provided an algorithm to decide parametric integer programming, i.e., the sentences of the form "$\forall b \in Q/\mathbb{Z}^p \ \exists x \in \mathbb{Z}^n : \ Ax \leq b$", when p and n are fixed, describe an algorithm that, given an integral matrix A and an integral vector c, computes the maximum difference

$$\max\{c^{\mathrm{T}}x : Ax \leq b\} - \max\{c^{\mathrm{T}}x : Ax \leq b,\ x \text{ integral}\}$$

over all vectors b, for which the integer program is feasible, and runs in polynomial time if the number of variables x is fixed.

Exercise 2 (Integer programming infeasibility)
Show that a system $Ax \leq b$ has no integral solution if and only if there is a subsystem $A'x \leq b'$ of $Ax \leq b$ with at most 2^n inequalities that has no integral solution (n is the number of variables).

Exercise 3 (Infinite lattice width)
Prove that a parametric polyhedron $P_b = \{x : Ax \leq b\}$ has finite lattice width if and only if the polyhedron $P_0 = \{x : Ax \leq 0\}$ has finite lattice width.