Integer Points in Polyhedra

Spring 2009

Assignment Sheet 10

Exercise 1 (Integer projections of polyhedra)
Show that the following sets can be viewed as integer projections of some polyhedra:

(a) A polyhedron $P \subseteq \mathbb{R}^n$.

(b) The set of integral points of a polyhedron $P \cap \mathbb{Z}^n$.

(c) An integer cone $\{\sum_{i=1}^{k} \lambda_i a_i : \lambda_i \geq 0 \text{ integer}\}$, where $a_1, a_2, \ldots, a_k$ are given integral vectors.

Exercise 2 (Mixed-integer programming)
Let A and B be matrices and b a vector. Describe an algorithm that finds a point (x, y) such that x is integral and $Ax + By \leq b$, and runs in polynomial time if the number of x-variables is fixed.

Exercise 4 (Hilbert bases)
Integral vectors $a_1, a_2, \ldots, a_k \in \mathbb{Z}^n$ are said to form a *Hilbert basis* if every integral vector $b \in \text{cone}(a_1, a_2, \ldots, a_k) \in \mathbb{Z}^n$ can be expressed as $b = \sum_{i=1}^{k} \lambda_i a_i$ with λ_i's being integral.

(a) Show that any rational cone can be generated by some Hilbert basis.

(b) Show that for any pointed rational cone, there is the unique minimal (with respect to inclusion) Hilbert basis.