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Exercises

Approximation Algorithms

Spring 2010

Sheet 5

Exercise 1

Consider MULTI CONSTRAINT KNAPSACK where n objects with profits $p_i \in \mathbb{Q}_+$ and budget requirement $a_i^j \in [0,1]$ are given. Let

$$OPT = \max_{I \subseteq \{1,\dots,n\}} \Big\{ \sum_{i \in I} p_i \mid \sum_{i \in I} a_i^j \leq 1 \; \forall j = 1,\dots,k \Big\}$$

(since after scaling we may assume that the available budgets are all 1). We consider the number k of budgets to be a fixed constant. Show that for any $\varepsilon > 0$ one can compute in time polynomial in n and $1/\varepsilon$ a solution $I \subseteq \{1,\dots,n\}$ with profit $\sum_{i \in I} p_i \geq OPT$ which is *nearly feasible*, i.e. $\sum_{i \in I} a_i^j \leq 1 + \varepsilon$ for all $j = 1,\dots,k$.

Hints: Find a suitable rounding + dynamic programming.

Exercise 2

For the BIN COVERING problem, we are given an instance $I = (a_1,\dots,a_n)$ with items $a_i \in [0,1]$ and aim at maximizing the number of bins, which are *covered* (that means the size of the assigned items is at least 1):

$$OPT = \max \Big\{ k \mid \exists I_1 \dot\cup \dots \dot\cup I_k = \{1,\dots,n\} : \forall j : \sum_{i \in I_j} a_i \geq 1 \Big\}$$

We assume that $a_i \geq \delta, i = 1,\dots,n$ for a constant $\delta > 0$. Give an asymptotic PTAS for this problem under the above assumption (i.e. a polynomial time algorithm that for any fixed $\varepsilon > 0$, covers at least $(1-\varepsilon)OPT - O(1)$ bins).

Hint: Adapt the APTAS of Fernandez de la Vega & Lueker from the lecture. In the grouping, you should round *down* the item sizes (instead of rounding them up).