Integer Points in Polyhedra

Spring 2009

Assignment Sheet 2

Exercise 1 (Characterization of lattices)
Prove that a set $\Lambda \subset \mathbb{R}^n$ is a lattice if and only if it is a discrete subgroup of $\mathbb{R}^n$.

Exercise 2 (Minkowski's bounds on the shortest vector)
Let $\Lambda \subset \mathbb{R}^n$ be a full-dimensional lattice. Using the Minkowski's theorem, derive upper bounds for the shortest vectors in $\Lambda \setminus \{0\}$ with respect to l_1-norm, l_2-norm, and l_∞-norm.

Exercise 3 (Dirichlet's theorem)
Let α be a real number and let $Q > 0$ be an integer. Show that there are integers p and q, where $0 < q \leq Q$, such that

$$\left|\frac{p}{q} - \alpha\right| \leq \frac{1}{qQ}.$$

Exercise 4 (Lagrange's four square theorem)
Prove that every positive integer n can be expressed as the sum of four squares,

$$n = x_1^2 + x_2^2 + x_3^2 + x_4^2,$$

where x_i's are integers.