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Location: ELD120 Discussion: 3.03.10

Exercises

Approximation Algorithms

Spring 2010

Sheet 1

Exercise 1
Give family of undirected graphs $G = (V,E)$ and terminals R, such that asymptotically (i.e. for $|V| \to \infty$) the Minimum spanning tree is a factor 2 more expensive then the cheapest Steiner tree.

Exercise 2
For the STEINER TREE problem, we are given an undirected weighted graph $G = (V,E)$ and a set of terminals $R \subseteq V$. It is the goal to find a tree T that connects all terminals. There exists a constant $c_0 > 1$ such that the following gap version of the 3-SET COVER problem is **NP**-hard:

> Given sets $S_1,\dots,S_m \subseteq \{1,\dots,n\}$ with $|S_i| = 3$ and a parameter $k \in \mathbb{N}$, distinguish
>
> - YES: There is a cover with $\leq k$ sets
> - NO: There is no cover with $\leq c \cdot k$ sets

Show that STEINER TREE is **APX**-hard, i.e. show that there is a constant $c_1 > 1$ such that finding a c_1-approximate STEINER TREE is **NP**-hard.
Hint: Construct a STEINER TREE instance with 1 terminal for each element, 1 Steiner node per set and 1 special root terminal (unit cost edges should suffice).

Exercise 3
Consider the MAXIMUM COVERAGE problem: Given sets $S_1,\dots,S_m$ over a universe of elements $U = \{1,\dots,n\} = \bigcup_{i=1}^m S_i$ and a parameter $k \in \mathbb{N}$. Choose k sets that cover as many elements as possible, i.e.

$$OPT := \max\Big\{|\bigcup_{i\in I} S_i| : |I| = k\Big\}$$

Show that a straightforward greedy algorithm gives a $\frac{e}{e-1} \approx 1.58$-approximation.