École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

May 19, 2017

Discrete Optimization (Spring 2017)

Assignment 11

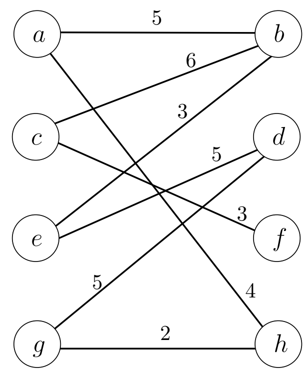
Problem 2 can be submitted until May 26 12:00 noon into the right box in front of MA C1 563.
You are allowed to submit your solutions in groups of at most three students.

Problem 1

Given the weighted graph on the right, find the following:

a) A matching that is not perfect and has weight 15.

b) A w-vertex cover of weight 16 where at least seven vertices have non zero weights.



Problem 2 ($\star$)

Let G be a graph and let A be its node-edge incidence matrix. We have seen in class that if G is bipartite then A is totally unimodular. Prove the converse, *i.e.*, if A is totally unimodular then G is bipartite.

Problem 3

Consider a bipartite graph $G = (U \cup W, E)$. Assume there exist matchings M_U and M_W covering vertices $U_1 \subseteq U$ and $W_1 \subseteq W$, respectively. Prove that there always exists a matching that covers $U_1 \cup W_1$.
Hint: The symmetric difference $M_U \Delta M_W$ consists of only cycles and paths.

Problem 4

Given a graph $G = (V, E)$, a subset $S \subseteq V$ is called *stable* (or *independent*) if $|e \cap S| \leq 1$ for each $e \in E$. The independent set problem (ISP) is to find a maximum cardinality stable set on G.
We know that an optimal (but not-necessarily integral) solution of a linear program (LP) can be found in polynomial time. Show that the ISP can be solved in polynomial time if G is bipartite.

a) Let $\bar{A}$, $\bar{b}$, $\bar{c}$ and x^* be given as the input, where x^* is an optimal solution of $\max\{\bar{c}^T x : x \in P\}$ and $P = \{x \in \mathbb{R}^n : \bar{A}x \leq \bar{b}\}$. If x^* is not a vertex of P, argue that one can find a vertex $\bar{x}$ of P in polynomial time such that $\bar{c}^T x^* = \bar{c}^T \bar{x}$.

b) Formulate an LP relaxation of the ISP in the form

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

such that $A \in \mathbb{Z}^{m \times n}$ is totally unimodular for bipartite G, $b \in \mathbb{Z}^m$ and the corresponding polytope is a convex hull of indicator vectors of stable sets on G. Additionally, require that the encoding length of A, b and c is polynomial in $|V|$ and $|E|$.