École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand
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December 2, 2016

Combinatorial Optimization (Fall 2016)

Assignment 9

Deadline: December 9 10:00, into the right box in front of MA C1 563.

Exercises marked with a $\star$ can be handed in for bonus points.

Problem 1

Give an example of two matroids $M_1(E, \mathcal{I}_1), M_2(E, \mathcal{I}_2)$ such that $(E, \mathcal{I}_1 \cap \mathcal{I}_2)$ is not a matroid.

Problem 2

Assume E is a finite set and $r : 2^E \to \mathbb{N}$ is a function that satisfies:

(i) $r(A) \leq |A|$ for any $A \subseteq E$.

(ii) If $A \subseteq B \subseteq E$, $r(A) \leq r(B)$.

(iii) r is submodular.

Show that $M(E, \mathcal{I})$ is a matroid, where $\mathcal{I} = \{A \subseteq E : r(A) = |A|\}$.

Problem 3 ($\star$)

Given an undirected graph $G(V, E)$ and $s, t \in V$, consider the problem of deciding whether there is an $s - t$ path that contains all the vertices. Show that this problem can be formulated as the intersection of three matroids (i.e. give three matroids such that such $s - t$ path exists if and only if there exists a set of a certain cardinality which is independent in all three matroids).