École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

April 13, 2018

Discrete Optimization (Spring 2018)

Assignment 7

Problem 3 can be **submitted** until April 20 12:00 noon into the box in front of MA C1 563. You are allowed to submit your solutions in groups of at most three students.

Problem 1

Determine the dual program for the following linear program:

$$\begin{array}{lrcl} \min & 3x_1 + 2x_2 - 3x_3 + 4x_4 & & \\ & 2x_1 - 2x_2 + 3x_3 + 4x_4 & \leq & 3 \\ & x_2 + 3x_3 + 4x_4 & \geq & -5 \\ & 2x_1 - 3x_2 - 7x_3 - 4x_4 & = & 2 \\ & x_1 \geq 0 & & \\ & x_4 \leq 0 & & \end{array}$$

Problem 2

In the GitHub repository of the course, in the 'Programming' folder, you will find the file 'Simplex.py' which contains the code to run the simplex algorithm on non-degenerate linear programs in standard form. However, some code is missing. Complete the code by filling the gaps (indicated by underscores) and send it to igor@malinovic.epfl.ch.

Problem 3 ($\star$)

Suppose you are given an oracle algorithm, which for a given polyhedron

$$P = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} \ : \ \tilde{A}\tilde{x} \leq \tilde{b}\}$$

gives you a feasible solution or asserts that there is none.
Consider the LP $\max\{c^T x \colon Ax \leq b,\, x \in \mathbb{R}^n\}$, and assume it is feasible and bounded. Show that one can obtain an optimal solution of the LP using a single call of the oracle algorithm on a suitable polyhedron. *Hint: Use duality theory!*

Problem 4

Consider the following linear program:

$$\begin{array}{rc} \max & x_1 + x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 6 \\ & x_1 + 2x_2 \leq 8 \\ & 3x_1 + 4x_2 \leq 22 \\ & x_1 + 5x_2 \leq 23 \end{array}$$

Show that $(4/3, 10/3)$ is an optimal solution by using weak duality.

Problem 5

Let $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ a bounded, non-empty polyhedron. Formulate a linear program that computes the largest ball inside P.