École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

March 30, 2018

Discrete Optimization (Spring 2018)

Assignment 6

Problem 5 can be **submitted** until April 13 12:00 noon into the box in front of MA C1 563. You are allowed to submit your solutions in groups of at most three students.

Problem 1

Let $P = \{x \in \mathbb{R}^n \colon Ax \leq b\}$ be a polyhedron. Show that the following are equivalent:

i) x^* is a vertex of P.

ii) x^* is a basic feasible solution.

iii) For every feasible $x_1, x_2 \neq x^* \in P$ one has $x^* \notin \operatorname{conv}\{x_1, x_2\}$.

Problem 2

Prove or give a counter-example for the following statements:

i) Let B be an optimal basis. If λ_B is strictly positive then the optimal solution is unique.

ii) If the optimal solution is unique then λ_B is strictly positive for the optimal basis B.

Problem 3

Prove that the truthfulness of the statement in Problem 2.ii) changes if we assume that the considered polyhedron is non-degenerate.

Problem 4

Assume you are the production manager for a lecture of an online course. For the production, n tasks have to be executed. A task j requires a working time of p_j hours to be completed. You have m employees at your disposal that can each, due to his or her qualifications, work on a subset of the tasks. Denote by S_i the set of jobs that employee i can work on.
As the production manager you want to create a work allocation plan that ensures that all tasks are completed. However, this allocation should also be fair. Consider the maximum number of working hours of each employee. You would like to minimize this quantity.
Model this problem as a linear program.

Problem 5 ($\star$)

Consider the following linear program:

$$
\begin{array}{rlll}
\max & 6a + 9b + 2c & & \\
\text{subject to} & a + 3b + c & \leq -4 & (1) \\
& b + c & \leq -1 & (2) \\
& 3a + 3b - c & \leq 1 & (3) \\
& a & \leq 0 & (4) \\
& b & \leq 0 & (5) \\
& c & \leq 0 & (6)
\end{array}
$$

Solve the linear program with the Simplex method and initial vertex $(-1, -1, 0)^T$. For each iteration indicate all the parameters as in the previous exercise including the optimal value and the proof of optimality.