École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

March 31, 2017

Discrete Optimization (Spring 2017)

Assignment 6

Problem 4 can be **submitted** until April 7 12:00 noon into the right box in front of MA C1 563.
You are allowed to submit your solutions in groups of at most three students.

Problem 1

For each of the following assertions, provide a proof or a counterexample.

i) An index that has just left the basis B in the simplex algorithm cannot enter in the very next iteration.

ii) An index that has just entered the basis B in the simplex algorithm cannot leave again in the very next iteration.

Problem 2

Let $P = \{x \in \mathbb{R}^n \colon Ax \leq b\}$ be a polyhedron. Show that the following are equivalent:

i) x^* is a vertex of P.

ii) x^* is a basic feasible solution.

iii) For every feasible $x_1, x_2 \neq x^* \in P$ one has $x^* \notin \operatorname{conv}\{x_1, x_2\}$.

Problem 3

Recall the linear program from the last assignment:

$$
\begin{aligned}
\max \quad & a + 3b & \\
\text{s.t.} \quad & a + b \leq 2 & (1)\\
& a \leq 1 & (2)\\
& -a \leq 0 & (3)\\
& -b \leq 0 & (4)
\end{aligned}
$$

Solve it with the Simplex method starting with the initial feasible basic solution induced by the constraints (2) and (4). For each iteration indicate the current basis and the corresponding vertex, λ_B, the direction in which the Simplex moves and how far it moves. At the end indicate the optimal objective value and the proof of optimality (i.e. the final λ).

Problem 4 ($\star$)

Consider the following linear program:

$$
\begin{aligned}
\max \quad & 6a + 9b + 2c & \\
\text{subject to} \quad & a + 3b + c \leq -4 & (1)\\
& b + c \leq -1 & (2)\\
& 3a + 3b - c \leq 1 & (3)\\
& a \leq 0 & (4)\\
& b \leq 0 & (5)\\
& c \leq 0 & (6)
\end{aligned}
$$

Solve the linear program with the Simplex method and initial vertex $(-1,-1,0)^T$. For each iteration indicate all the parameters as in the previous exercise including the optimal value and the proof of optimality.