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Discussions from: December 5, 2010

Combinatorial Optimization

Fall 2010

Assignment Sheet 6

Exercise 1

Let $G = (V, E)$ be an undirected graph. We define $\mathscr{I}$ as the collection of those subsets $I \subseteq V$ of vertices that can be covered by a matching in G (this means that G has a perfect matching iff $\mathscr{I}$ is the collection of *all* subsets of vertices). Prove that $(V, \mathscr{I})$ is a matroid.

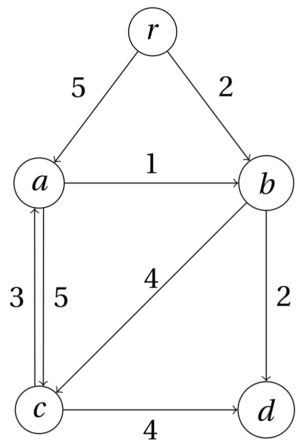
How does this fact relate to the problem of computing maximum weight matchings?

Exercise 2 (⋆)

Let $D = (V, A)$ be a digraph, let N be its incidence matrix, and let M be the linear matroid defined by N (that is, the elements of M are the columns of N, and its independent sets are exactly the linearly independent sets of columns of N). Prove that M is the forest matroid of the undirected graph underlying D.

Exercise 3

Trace the steps of algorithm from the lecture to compute a minimum weight arborescence rooted at r in the following example.



Exercise 4
Let $D = (V, A)$ be a directed graph. A *branching* in D is a subset $B \subset A$ of arcs such that the underlying undirected graph is a forest and each vertex $v \in V$ has at most one incoming arc.

1. Let $\mathscr{B}$ be the set of all branchings in D. Prove that $(A, \mathscr{B})$ is the intersection of two matroids.

2. Let $r \in V$. Show how to model the arborescences rooted at r using the intersection of two matroids.

Exercise 5 ($\star$)
Let $D = (V, A)$ be a directed graph with root $r \in V$. Suppose that D does not contain an arborescence rooted at r. Prove that there exists a strongly connected component K in D such that $r \notin K$ and $|\delta^{in}(K)| = 0$.