École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

March 24, 2017

Discrete Optimization (Spring 2017)

Assignment 5

Problem 3 can be **submitted** until March 31 12:00 noon into the right box in front of MA C1 563.
You are allowed to submit your solutions in groups of at most three students.

Problem 1

Consider the following linear program:

$$\begin{array}{rr} \max & a + 3b \\ \text{s.t.} & a + b \leq 2 \\ & a \leq 1 \\ & -a \leq 0 \\ & -b \leq 0 \end{array}$$

Compute the optimal solution via vertex enumeration. Give an alternative proof that the vertex you found is an optimal solution.

Problem 2

A polyhedron $P = \{x \in \mathbb{R}^n \colon Ax \leq b\}$ *contains a line*, if there exists a nonzero $v \in \mathbb{R}^n$ and an $x^* \in \mathbb{R}^n$ such that for all $\lambda \in \mathbb{R}$, the point $x^* + \lambda \cdot v \in P$. Show that a nonempty polyhedron P contains a line if and only if A does not have full column-rank.

Problem 3 ($\star$)

Suppose you are given an oracle algorithm, which for a given polyhedron

$$P = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} \ : \ \tilde{A}\tilde{x} \leq \tilde{b}\}$$

gives you a feasible solution or asserts that there is none. Show that using a *single* call of this oracle one can obtain an *optimum* solution for the LP

$$\max\{c^T x \ : \ x \in \mathbb{R}^n; \ Ax \leq b\},$$

assuming that the LP is feasible and bounded.
Hint: Use duality theory!

Problem 4

Consider the following linear program:

$$\begin{array}{rr} \max & x_1 + x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 6 \\ & x_1 + 2x_2 \leq 8 \\ & 3x_1 + 4x_2 \leq 22 \\ & x_1 + 5x_2 \leq 23 \end{array}$$

Show that $(4/3, 10/3)$ is an optimal solution by using strong duality.

Problem 5

Let $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ a bounded, non-empty polyhedron. Formulate a linear program that computes the largest ball inside P.