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Discussions from: November 11, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 9

$\star$ exercises can be handed in for bonus points. Due date is Friday November 20.

Let $G = (V, E)$ be a connected graph.

Exercise 1
Let M_1, M_2 be two maximal matchings in G. Prove that $|M_1| \leq 2|M_2|$. (Recall that a matching of G is maximal if it is not properly contained in any other matching of G.).

Exercise 2
A matching M of G is perfect if it covers every vertex of G. Prove that if G is a tree then it has at most one perfect matching.

Exercise 3
Consider the 8×8 chessboard. Can you use tiles of size 2×1 to cover all the squares of the board except two diagonally opposite corners? [Hint: think of a perfect matching in a bipartite graph].

Exercise 4 ($\star$)

(i) Prove that if M_1 and M_2 are matchings of G and $|M_2| > |M_1|$ then there exists at least $|M_2| - |M_1|$ vertex-disjoint M_1-augmenting paths.

(ii) Prove that if M is a matching of G that is not maximum cardinality then there exists a maximum cardinality matching M^* such that every vertex covered by M is also covered by M^*. [Hint: use part (i)]