
Combinatorial Optimization

Fall 2015

Assignment Sheet 8

★ exercises can be handed in for bonus points. Due date is Friday November 13.

Exercise 1

Show that the problem of computing the longest path in an undirected graph can be formulated as the intersection of three matroids.

Exercise 2 (★)

Let $P \subseteq \mathbb{R}^n$ be a full dimensional polytope that contains the origin in its interior. Let $x \in \mathbb{R}^n$. Prove that x is a vertex of P if and only if $\{y \in \mathbb{R}^n \mid x^\top y \leq 1\}$ defines a facet of P^0 .

Exercise 3

Let C be the square with corners at $(1, 1), (1, -1), (-1, 1), (-1, -1)$. Draw the polar of C .

Exercise 4

Let $G = (V, E)$ be a graph. The forest polytope associated with G is

$$P = \left\{ x \in \mathbb{R}_{\geq 0}^{|E|} : x(E[S]) \leq |S| - 1 \forall S \subseteq V, S \neq \emptyset \right\}$$

Assume you have access to an oracle that given an edge weighted directed graph H and two nodes $s, t \in V(H)$ finds a minimum weight directed $s - t$ cut in H . Show how to solve the separation problem for P in polynomial time.