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Discussions from: October 21, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 6

Exercises marked with $\star$ can be handed in for bonus points. Due date is Friday October 30.

Exercise 1
Let $G = (V, E)$ be an undirected graph. Show that the function $f : 2^V \to \mathbb{R}$ defined as $f(U) = |\delta(U)|$ for each $U \subseteq V$ is submodular. Recall that $\delta(U)$ is the set of edges in E that have one endpoint in U and the other in $V \setminus U$.

Exercise 2
Show that a function $f : 2^U \to \mathbb{R}$ is submodular if and only if for all $A \subseteq B \subseteq U$ and $e \in U \setminus B$ we have

$$f(A+e) - f(A) \geq f(B+e) - f(B)$$

Exercise 3
Let $M = (X, \mathscr{I})$ be a matroid. Show that for any $Z \in \mathscr{I}$ there exists $H \subseteq X$ with $Z \cup H \in \mathscr{I}$ and $|Z \cup H| = r_M(X)$.

Exercise 4
In the lecture today we used the fact that two polytopes P and Q are equal if and only if for every vector c we have

$$\max_{x \in P} c^\top x = \max_{y \in Q} c^\top y$$

Provide a proof for this statement.

Exercise 5 ($\star$)
Suppose that we are given two polytopes $P_1 = \{x \in \mathbb{R}^n | A_1 x \geq b_1\}$ and $P_2 = \{x \in \mathbb{R}^n | A_2 x \geq b_2\}$ and we want to check whether $P_1 \subseteq P_2$. Show how this can be done using linear programming.