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Discussions from: October 14, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 5

Exercises marked with $\star$ can be handed in for bonus points. Due date is Friday October 23.

Exercise 1
Prove that a spanning connected subgraph of G is a spanning tree if and only if it has $|V|-1$ edges.

Exercise 2
In the MST problem we want to find a spanning tree $H=(V,T)$ of G that minimizes $\sum_{e\in T} w(e)$. Suppose we want to minimize instead $\max_{e\in T} w(e)$. Prove that if $H=(V,T)$ is an MST of G then H is also an optimal solution for our new problem.

Exercise 3
Let X be a finite set and $\mathscr{I}$ a collection of subsets of X satisfying

(i) $\emptyset \in \mathscr{I}$

(ii) If $Y \in \mathscr{I}$ and $Z \subseteq Y$ then $Z \in \mathscr{I}$.

Show that the following two conditions are equivalent

1. If $Y, Z \in \mathscr{I}$ and $|Y| < |Z|$ then $Y \cup \{x\} \in \mathscr{I}$ for some $x \in Z \setminus Y$
2. For any subset Y of X any two bases of Y have the same cardinality.

Exercise 4 ($\star$)
Let $G=(V,E)$ be a graph.

(i) Let $\mathscr{I} = \left\{M \subseteq E : M \text{ is a matching}\right\}$. Show that $(E,\mathscr{I})$ is not a matroid.

(ii) Let $\mathscr{I}' \subseteq 2^V$ be defined as follows: for $U \subseteq V$ we have $U \in \mathscr{I}'$ if and only if there exists a matching in G that covers all vertices of U. Show that $(V,\mathscr{I}')$ is a matroid.