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Discussions from: October 7, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 4

Exercises marked with ⋆ can be handed in for bonus points. Due date is Friday October 16.

Exercise 1
Let $A \in \mathbb{R}^{n\times n}$ be an invertible matrix and $b \in \mathbb{R}^n$ a vector. In class we defined the ellipsoid $E(A,b)$ as the imagine of the unit ball under the linear mapping $t(x) = Ax + b$. Show that

$$E(A,b) = \left\{x \in \mathbb{R}^n : (x-b)^\top A^{-\top} A^{-1}(x-b) \leq 1\right\}$$

Exercise 2
Draw $E(A,b)$ for $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Exercise 3
Show that the unit simplex $\Delta = \text{conv}\{0, e_1, \cdots, e_n\} \subset \mathbb{R}^n$ has volume $\frac{1}{n!}$.

Exercise 4 (⋆)
Let $P = \left\{x \in \mathbb{R}^n : Ax \leq b\right\}$ be a full dimensional 0/1 polytope and $c \in \mathbb{Z}^n$. We will show how we can use the ellipsoid method to solve the optimization problem $\max\left\{c^\top x : x \in P\right\}$.

Define $z^* := \max\left\{c^\top x : x \in P\right\}$ and $c_{\max} := \max\{|c_i| : 1 \leq i \leq n\}$.

(i) Show that the ellipsoid method requires $O(n^3 \log(n) c_{max})$ iterations to decide whether $P \cap (c^\top x \geq \beta) = \emptyset$ for some integer β. (Find a suitable initial ellipsoid and stopping value L)

(ii) Show that we can use binary search to find z^* with $\log(nc_{\max})$ calls to the ellipsoid method.

(iii) Show how you can find an optimal solution x^* such that $c^\top x^* = z^*$ in polynomial time.