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Discussions from: December 9, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 13

⋆ exercises can be handed in for bonus points. Due date is Friday December 18.

Given $a, c \in \mathbb{Z}^n$ and $b \in \mathbb{Z}$ recall the maximum knapsack problem:

$$
\begin{aligned}
(P) \quad & \max c^\top x \\
& a^\top x \leq b \\
& x \in \{0,1\}^n
\end{aligned}
$$

where we can assume wlog that $c, a \geq 0$ and $||a||_\infty \leq b$.

Exercise 1

Apply the algorithm seen in class to the following instance.

$$
\begin{aligned}
\max \quad & x_1 + 2x_2 + 2x_3 + 3x_4 \\
\text{s.t.} \quad & 3x_1 + 5x_2 + 7x_3 + 6x_4 \leq 14 \\
& x \in \{0,1\}^4
\end{aligned}
$$

Exercise 2

Consider the modified problem

$$
\begin{aligned}
(P') \quad & \max \hat{c}^\top x \\
& a^\top x \leq b \\
& x \in \{0,1\}^n
\end{aligned}
$$

where $\hat{c}_i = \lfloor c_i / M \rfloor$ for all $i \in \{1, \cdots, n\}$ for some $M \in \mathbb{R}_{\geq 0}$. Let x' be an optimal solution to P'. Give an example where $cx' < ||c||_\infty$.

Exercise 3 (⋆)

Show that one can solve P in time polynomial in n and $||a||_\infty$. (Hint: formulate as a longest path problem in an acyclic graph).

Exercise 4

Show that for each $n \in \mathbb{N}$, there is a graph with $2n$ nodes where the algorithm seen in class for vertex cover gives a solution that is exactly 2-approximated.