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Discussions from: December 2, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 12

⋆ exercises can be handed in for bonus points. Due date is Friday December 11.

Exercise 1 (⋆)
Recall the following definition

$$L_K = \{(a,\beta) : a \in \mathbb{Z}^n, \beta \in \mathbb{Z} : \exists x \in \{0,1\}^n : a^T x = \beta\}$$
$$L_{IP_=} = \{(A,b) : A \in \mathbb{Z}^{m\times n}, b \in \mathbb{Z}^m : \exists x \in \{0,1\}^n : Ax = b\}$$

In this exercise you will show that there exists a polynomial-time reduction from $L_{IP_=}$ to L_K while we have seen a randomized reduction in class.

(i) Given two equations $a_1 x = b_1$ and $a_2 x = b_2$ with $a_1, a_2 \in \{0,1\}^n$, choose $\lambda \in \mathbb{Z}^2$ such that

$$\{x \in \{0,1\}^n : a_1 x = b_1 \text{ and } a_2 x = b_2\} = \{x \in \{0,1\}^n : \lambda_1(a_1 x) + \lambda_2(a_2 x) = \lambda_1 b_1 + \lambda_2 b_2\}$$

(ii) Provide a deterministic polynomial-time reduction from $L_{IP_=}$ to L_K. You may assume that A is a matrix with entries in $\{0,1\}$ only and that b is the all-ones vector.

Exercise 2
Consider the following variation of 3-SAT

- **1-in-3-SAT:** given a boolean formula F in conjunctive normal form where each clause has 3 literals, is there a truth assignment to the variables of F such that each clause of F has exactly one true literal?

Prove that 1-in-3-SAT is NP-complete.

Exercise 3
The decision problem of **3D Matching** is defined as

- Given three disjoint sets X, Y and Z, and a set of triples $E \subseteq X \times Y \times Z$, does E contain a matching? (i.e. is there a subset $M \subseteq E$, such that each element of X, Y and Z appears in exactly one triple of M?)

Show that 3D-matching is NP-complete using a reduction from 3-SAT.