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Discussions from: November 18, 2015

Combinatorial Optimization

Fall 2015

Assignment Sheet 10

⋆ exercises can be handed in for bonus points. Due date is Friday November 27.

Exercise 1
Let $P = \left\{x \in \mathbb{R}^b : Ax \le b\right\}$ be a polehydron and $c^\top x \le \delta$ a valid inequality for P that defines the face F. Then by linear programming duality we have

$$\delta = \max\left\{c^\top x : Ax \le b\right\} = \min\left\{b^\top \lambda : A^\top \lambda = c, \lambda \ge 0\right\}$$

and there exists a $\lambda \in \mathbb{R}^m_{\ge 0}$ such that $c = \lambda^\top A$ and $\delta = \lambda^\top b$. Let $I \subseteq \{1, \cdots, m\}$ be the set of indices with $\lambda_i > 0$ and $A_I x \le b_I$ the subsystem of $Ax \le b$ which corresponds to the indices in I. Show that

$$F = \{x \in P : A_I x = b_I\}.$$

Exercise 2
Let $P = \left\{x \in \mathbb{R}^b : Ax \le b\right\}$ be a polehydron and $A^+ x \le b^+$ the subsystem corresponding to the implicit inequalities. Show that we can always find a point $\overline{x} \in P$ such that $A^+ \overline{x} < b^+$.

Exercise 3
Let $P \subseteq \mathbb{R}^n$ be a full-dimensional polyhedron and F_1, F_2 be facets of P described by the inequalities $ax \le \beta$ and $cx \le \delta$ respectively. Show that $F_1 = F_2$ if and only if $ax \le \beta$ and $cx \le \delta$ are equivalent inequalities (i.e. one is a positive scalar multiple of the other).

Exercise 4 (⋆)
Let P_M be the matching polyhedron and $w^\top x \le \gamma$ a facet defining inequality of P_M. Suppose that $w \ge 0$ and let $G^* = (V^*, E^*)$ be the subgraph of G induced by the edges e with $w(e) > 0$. Prove that G^* is connected. [Hint: fill in the missing details from the proof given in class].