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Due Date: November 17, 2008

Combinatorial Optimization

Fall 2008

Assignment Sheet 4

Exercise 1 (Linearity space)
Prove that each non-empty polyhedron $P \subseteq \mathbb{R}^n$ can be represented as $P = L + Q$, where $L \subseteq \mathbb{R}^n$ is a linear space and $Q \subseteq \mathbb{R}^n$ is a pointed polyhedron.

Exercise 2 (Dimension of a polyhedron)
Let $A \in \mathbb{R}^{m\times n}$ be a matrix and $b \in \mathbb{R}^m$ a vector. Let $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ be the polyhedron defined by A and b. Prove that $\dim(P) = n - \operatorname{rank}(A^=)$, where $A^= x \leq b^=$ is a subsystem of $Ax \leq b$ consisting of implicit equalities.

Exercise 3 (Dimension of a minimal face)
Show that the dimension of each minimal face of a polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ is equal to $n - \operatorname{rank}(A)$.

Exercise 4 (Pointed polyhedron)
Prove that a polyhedron has a vertex if and only if it does not contain a straight line.