PhD Doctoral Course – Network Design – 3th November 2009

8th Assignment

1. Construct an instance of multi-commodity flow on an undirected graph, where the cut-condition is not violated but a multi-commodity flow satisfying the given demands does not exist.

2. Construct an instance of two-commodity flow on an undirected graph, where there exists an half-integral multi-commodity flow satisfying the given demands, but not an integral one.

3. Construct an example showing that Hu's two-commodity flow theorem does not hold on directed graphs. Recall that the cut-condition on directed graph is:

$$\forall X \subseteq V, \sum_{i: s_i \in X, t_i \notin X} d_i \leq u(\delta^+(X)).$$

4. Let $G(V,E)$ be an undirected graph with capacity $u: E \to Q_+$ and let $\{s_1,t_1\},\ldots,\{s_k,t_k\}$ be pairs of vertices such that there exist a two vertices intersecting each $\{s_i,t_i\}$. Finally, let $d_1,\ldots,d_k \in Q_+$ be a set of demands. Show that the cut-condition implies the existence of a multi-commodity flow satisfying the given demand d_i for each $\{s_i,t_i\}$.

5. Prove/Disprove. Let $G(V,E)$ be an undirected graph with capacity $u: E \to Q_+$. Let $S = \{s_1,\ldots,s_k\}$ and $T = \{t_1,\ldots,t_k\}$ be vertices in V. Suppose that there exists a feasible multi-commodity flow for any set of demands corresponding to a requirement graph R that is a bipartite matching on the vertices $S \cup T$. Then, there exists a feasible multi-commodity flow for any set of demands corresponding to a requirement graph R that is any (non-bipartite) matching on the vertices $S \cup T$.