PhD Doctoral Course – Network Design – 20th October 2009

6th Assignment

1. Let $G(V,E)$ be an undirected graph. A function $f : 2^V \to \mathbb{Z}$ is *submodular* if $f(V) = 0$, and for every two sets $A,B \subseteq V$ *both* the following two conditions hold:

- $f(A) + f(B) \geq f(A \cup B) + f(A \cap B)$
- $f(A) + f(B) \geq f(A - B) + f(B - A)$

Show that the function $|\delta_G(\cdot)|$ is submodular.

2. A function $f : 2^V \to \mathbb{Z}$ is *weakly supermodular* if $f(V) = 0$, and for every two sets $A,B \subseteq V$ *at least one* of the following two conditions hold:

- $f(A) + f(B) \leq f(A \cup B) + f(A \cap B)$
- $f(A) + f(B) \leq f(A - B) + f(B - A)$

Let H be a subgraph of G. Show that, if f is a weakly supermodular function, then $f' : 2^V \to \mathbb{Z}$ is also a weakly supermodular function, where $f'(S) := f(S) - |\delta_H(S)|$, $\forall S \subseteq V$.

3. Consider the Generalized Steiner Network problem, and the LP shown in the lecture. Prove that, *at each iteration* of the LP-rounding algorithm, you can solve the LP in polynomial time since there is a polynomial time separation oracle.

4. Show that the class of weakly supermodular functions contains both the classes of downwards monotone functions and proper functions.