PhD Doctoral Course – Network Design – 13th October 2009

5th Assignment

1. Give a family of instances of the metric Facility Location problem where the ratio between the cost of a solution output by the primal-dual Algorithm, and the cost of the optimal solution tends to 3.

2. In phase 2 of the primal-dual algorithm, instead of picking all the special edges in the graph T, pick all the tight edges. Does the proof of Lemma 2 (shown in the lecture) still hold?

3. Consider the following generalization to arbitrary demands. For each client j, a non-negative demand d_j is specified, and any open facility can serve this demand. The cost of serving this demand via facility i is $c_{ij}d_j$. Give an IP and LP-relaxation for this problem and extend the primal-dual algorithm shown in the lecture to get a 3-approximation algorithm for this case.

4. Consider the following modification to the metric Facility Location problem. Define the cost of connecting facility i to client j to be c_{ij}^2. Suppose that the costs c_{ij} satisfy triangle inequality (but the new costs c_{ij}^2 do not!). Show that the primal-dual algorithm shown in the lecture gives an approximation guarantee of factor 9 for this case.