Problem Set 3.

3-1: What is the number of *into* (that is, injective) maps from a set of n elements to a set of m elements, where $m \geq n$?

3-2: There are n married couples attending a dance. How many ways are there to form n pairs for dancing if no wife should dance with her husband?

3-3: (a) Determine the number of permutations with exactly one fixed point.
(b) Count the permutations with exactly k fixed points.

3-4: Which set of dominoes has larger cardinality:
- dominoes containing numbers from 0 to 8 and admitting doubles (that means, any number can appear twice on the same domino piece) or
- dominoes containing numbers from 0 to 9 without doubles (the two numbers appearing on the same domino piece must be distinct).

3-5: How many positive integers are there that divide 10^{40} or 20^{30}?

3-6: How many positive integers less or equal than 385 are there such that they are not divisible by neither of the following numbers: $5, 7, 11$?

3-7: Prove that every tree with a vertex of degree n has at least n vertices of degree one.

3-8: Prove that $\sum_{d|n} \varphi(d) = n$ for every natural number n, where the sum is taken over all natural numbers d dividing n.

3-9: Given 20 distinct positive integers, not greater than 70. Show that among their pairwise differences, at least four are equal.

3-10: * Let A and B be vertices of a graph such that any path from A to B contains at least 5 edges. Show that we we can assign a number between 1 and 5 to each edge, so that every path from A to B has an edge whose number is one, an edge whose number is two, ..., and edge, whose number is 5.