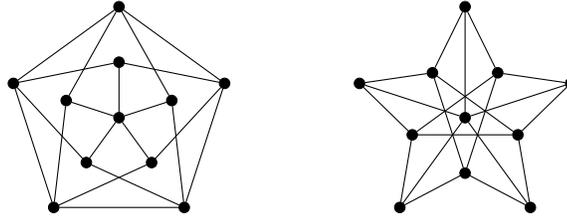


Graph theory - problem set 3

March 8, 2017

Exercises

1. For what values of n does the graph K_n contain an Euler trail? An Euler tour? A Hamilton path? A Hamilton cycle?
2. (a) For what values of m and n does the complete bipartite graph $K_{m,n}$ contain an Euler tour?
(b) Determine the length of the longest path and the longest cycle in $K_{m,n}$, for all m, n .
3. (a) Find a graph such that every vertex has even degree but there is no Euler tour.
(b) Find a disconnected graph that has an Euler tour.
4. Determine the girth and circumference of the following graphs.



Problems

5. Let G be a connected graph that has an Euler tour. Prove or disprove the following statements.
 - (a) If G is bipartite then it has an even number of edges.
 - (b) If G has an even number of vertices then it has an even number of edges.
 - (c) For edges e and f sharing a vertex, G has an Euler tour in which e and f appear consecutively.
6. Show that if $k > 0$ then the edge set of any connected graph with $2k$ odd-degree vertices can be split into k trails.
7. Prove that in any connected graph G , there is a walk that uses each edge exactly twice.
8. Let G be a connected graph with an even number of edges such that all the degrees are even. Prove that we can color each of the edges of G red or blue in such a way that every vertex has the same number of red and blue edges touching it.
- 9.* Prove that the edges of every graph G can be colored with red and blue in such a way that for every vertex $v \in V(G)$, the difference of the number of red edges and the number of blue edges touching v is at most 2.