

Graph theory - problem set 1

February 23, 2017

Exercises

- Is C_n a subgraph of K_n ?
 - For what values of n and m is $K_{n,n}$ a subgraph of K_m ?
 - For what n is C_n a subgraph of $K_{n,n}$?
- Given a graph G with vertex set $V = \{v_1, \dots, v_n\}$ we define the *degree sequence* of G to be the list $d(v_1), \dots, d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:
 - 3, 3, 2, 2, 2, 1
 - 6, 6, 6, 4, 4, 3, 3
- Construct two graphs that have the same degree sequence but are not isomorphic.
- A graph is *k-regular* if every vertex has degree k . Describe all 1-regular graphs and all 2-regular graphs.
- Prove that the number of odd-degree vertices in a graph is always even.
- How many (labelled) graphs exist on a given set of n vertices? How many of them contain exactly m edges?

Problems

- Do graphs with the following degree sequences exist:
 - 6, 6, 6, 4, 4, 2, 2
 - 6, 6, 6, 6, 5, 4, 2, 1?
- Let G be a graph with minimum degree $\delta > 1$. Prove that G contains a cycle of length at least $\delta + 1$.
- How many (labelled) graphs on the vertex set $\{1, \dots, n\}$ are isomorphic to P_n ? How many are isomorphic to C_n ?
- Show that every graph on at least two vertices contains two vertices of equal degree.
- For every $n \geq 6$, find a graph on n vertices, and $n + 2$ edges that contains exactly 6 cycles.
- What is the maximum number of edges in a bipartite graph on n vertices? (Prove your answer.)
- Let G be a graph containing a cycle C , and assume that G contains a path of length at least k between two vertices of C . Show that G contains a cycle of length at least \sqrt{k} .
 - *Prove the same statement with $\sqrt{2k}$ instead of \sqrt{k} .