

Packing and covering - problem set 9

April 15, 2014.

1. Use Blichfeldt's method to give an upper bound on $\delta(B^d)$ (the density of a packing of \mathbb{R}^d with unit spheres) using the following density function:

$$D(r) = \begin{cases} \frac{1}{d+1} & \text{if } r \leq \sqrt{2} - \epsilon, \\ 0 & \text{if } r > \sqrt{2} - \epsilon \end{cases}$$

where ϵ is a fixed small positive number (you also have to check that Blichfeldt's condition holds for $D(r)$).

2. Construct the following lattice packing of spheres in \mathbb{R}^3 : we take a regular triangular lattice in a plane, place touching spheres with these centers, and then we put the following layer above it so that the centers of the spheres above project down to the centerpoints of the triangles. We continue placing the subsequent layers in this manner. Find three vectors that generate this lattice, and calculate the density of this sphere packing.
3. Let κ_d denote the volume of the d -dimensional unit ball B^d , and let s_d denote the surface area of its boundary S^{d-1} . Prove that $s_d = d \kappa_d$.
4. Let C be a set of points in \mathbb{R}^d with no points of accumulation. We say that $C' \subseteq C$ is *Dirichlet* if the closed Dirichlet cells corresponding to the elements of C' have at least one point in common, and we say that $C' \subseteq C$ is *Dirichlet maximal* if there is no $C'' \subseteq C$ that is Dirichlet and such that $C' \subset C''$.

Prove that if C' is Dirichlet maximal, then there is a closed ball B containing no elements of C in its interior and such that $B \cap C = C'$.

- 5*. There are a hundred ants on a 1 m long rod, which hangs in the air. The ants are placed at random locations, and they are facing randomly either the left end, or the right end of the rod. At a given time, all of them start to move at once, with a speed of 1 meter per minute. If two ants meet, then both of them turn back and start moving in the opposite direction without loss of speed. Once an ant reaches the end of the rod, he falls down. How much time do we have to wait so that there are certainly no ants on the rod?