

Discrete mathematics - problem set 4

October 30, 2014.

1. Define a relation, with respect to which

$$\mathbb{Z} \times \mathbb{Z} = \{(x, y) : x, y \in \mathbb{Z}\}$$

becomes a partially ordered set, and prove that this set, with the relation chosen, becomes indeed a partially ordered set.

2. Find other examples of partially ordered sets apart from those given during the lectures and the one in the exercise above.
3. You are given a group of women. Between two women A and B , we define a relationship if A is the mother of B . Does the set of women, together with this relationship becomes a partially ordered set? Justify your answer!
4. Prove that every sequence of $mn + 1$ distinct numbers contains an increasing sequence of length $n + 1$ or a decreasing sequence of length $m + 1$.
5. Let $k \leq n/2$, and \mathcal{F} be a family of subsets of $\{1, 2, \dots, n\}$ such that every member of \mathcal{F} has size at most k , and no member of \mathcal{F} is contained in another member of \mathcal{F} . Prove that the cardinality of \mathcal{F} is at most $\binom{n}{k}$.
6. Let G be a bipartite graph with n vertices. Let m denote the maximum size of a matching in G . Prove that G has an independent set (that is, a set of pairwise non- adjacent vertices) of size $n - m$.
- 7*. Doctor Jekyll has created a new pill, whereof he claims, if used on daily basis, results in a daily decrease in body weight or in cholesterol levels, or, during some days, both together. Mr.Hyde started using the pill regularly for a month. However, at the end of the month both his weight and his cholesterol levels were the same as before. Did Dr.Jekyll lie about the properties of the pill? Justify your answer!