

## Packing and covering - problem set 7

April 2, 2014.

1. Let  $D(C)$  denote the difference region of  $C$ , as defined in Problem Set 4:  $D(C) = C + (-C)$ . For any set  $P \subseteq \mathbb{R}^2$ , show that  $\mathcal{C} = \{C + p | p \in P\}$  is a packing if and only if

$$\mathcal{D} = \left\{ \frac{D(C)}{2} + p | p \in P \right\}$$

is a packing.

**Definition.** Given  $n$  points  $O_1, \dots, O_n$  in  $\mathbb{R}^2$ , let  $D_i$  be the set of points in  $\mathbb{R}^2$ , which are at least as close to  $O_i$  as to any other  $O_j$ , i.e.,

$$D_i = \left\{ x \in \mathbb{R}^2 : \min_{1 \leq j \leq n} |x - O_j| = |x - O_i| \right\}, \quad 1 \leq i \leq n.$$

$D_i$  is called the *Dirichlet cell* or *Voronoi cell* of  $O_i$ .

2. Show that for any convex polygon  $D$  and points  $O_1, \dots, O_n \in D$ , the Voronoi region of each  $O_i$  intersected with  $D$  is a convex polygon.
3. Let  $\Lambda_1 = \mathbb{Z}^2$ , and  $\Lambda_2$  be the regular triangular lattice in the plane. Determine the corresponding Voronoi cells belonging to the points of the lattices. Determine the Voronoi cells with respect to the Manhattan distance as well (remember, it is defined by

$$d_1((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|.$$

4. Show that for every  $n \geq 4$ , there exists a set of  $n$  points of the plane, so that one of the corresponding Voronoi cells has exactly  $n - 1$  vertices.
5. Is it true that for any set of 10 points in the plane, there exists a corresponding Voronoi cell, which is bounded?
- 6\*. On a Saturday evening, you would like to go to a rendez-vous, but you loose your way at a junction: you don't know whether to turn left or right. There are two policemen at the junction, who know the city very well. However, they are a bit peculiar: one of them always tells the truth, whereas the other always lies, but you don't know which one is which. You are only allowed to ask one question (from one of them). What should be your question, so that you find the correct way?