

Discrete mathematics - problem set 3

October 2, 2014.

1. There are n married couples attending a dance. How many ways are there to form n pairs for dancing if no wife should dance with her husband?
2. (a) Determine the number of permutations with exactly one fixed point.
(b) Count the permutations with exactly k fixed points.
3. Which set of dominoes has larger cardinality:
 - dominoes containing numbers from 0 to 8 and admitting doubles (that means, any number can appear twice on the same domino piece) or
 - dominoes containing numbers from 0 to 9 without doubles (the two numbers appearing on the same domino piece must be distinct).
4. How many such positive integers are there that divide 10^{40} or 20^{30} ?
5. How many positive integers under 1000 are there such that they are not divisible by neither of the following numbers: 2, 3, 5, 7?
6. Prove every tree with at least two vertices contains at least two vertices of degree one.
7. Given a matrix such that the average of entries in each row is at least 10, show that there is a column in which the average of the entries is at least 10.
8. What is the number of *into* (that is, injective) maps from a set of n elements to a set of m elements, where $m \geq n$?
- 9*. Fred Flinstone along with 7 other friends from the Water Buffalo Order are participating in a Bedrock TV show, where they can win a prize of 1 million clams if they can solve the following challenge: a number from 0 to 7 will be stuck on the forehead of each of them (the same number may appear multiple times). No one can see his own number, but can see everyone else's number. Once they saw all the numbers (except their own), they are called into a room one by one, and asked to guess their own number. If at least one of the 8 participants can guess his own number, the team wins the prize. The rules are as follows: first, the participants agree on a strategy, but once the game has begun, there is no communication, they cannot send any signals to each other, and nobody can hear the guesses of the others. Can you help the Water Buffalo Order win the prize (Fred promised a beer for help)?