Geometric Graph Theory

Exercise session 10, April 29, 2015

Solutions of selected problems are accepted until May 6, 2015.

1. Show that if we color the edges of K_{3n-1} with two colors, then we can find n edges of the same color such that no two of them share a vertex.

2. Let G be a planar graph with n vertices. Let $f : V(G) \to (0, 1)$ be a so-called *weight function*, satisfying $\sum_{v \in V(G)} f(v) = 1$; that is, the total weight of the vertices of G is 1. Prove that the vertex set of G can be partitioned into three sets A, B, S such that $|S| \leq 4\sqrt{n}$, each of A, B has weight at most $\frac{9}{10}$, and there is no edge between A and B in G.
   Hint: you may proceed analogously as in the proof of the separator theorem at the lecture, but change the definition of the disc d. Figure out which parts of the proof can stay the same and which have to be modified.

✎ 3. Let G be a graph with n vertices. The *bisection width* of G is the minimum number of edges one has to remove from G so that the vertex set of the resulting graph G' can be divided into two parts, A and B, such that there is no edge between A and B in G' and $|A|, |B| \leq 2n/3$.
   Prove that there is a positive constant c such that for every positive integer m, the bisection width of the $m \times m$ grid is at least cm.

$ 4. Alice and Bob are traveling to Wonderland. They know from the Wonderland's ambassador that they have to go through the customs separately. Moreover, they will have to choose one package of chocolate and eat it before being allowed to enter. In Wonderland, they make milk and dark chocolate. One type is distributed in green packages and the other type in yellow; however, which color is used for which type remains a secret for outsiders. Alice likes milk chocolate, whereas Bob likes dark chocolate. If they discuss their plan before entering the border, do they have a chance better than 25% that both of them choose the chocolate they like?

✎ — an optional homework. You may submit a written solution of this problem until the beginning of the next lecture and receive our feedback.

$ — an optional contest problem. You may submit a solution until the beginning of the next lecture. We encourage you to participate.

New exercises and notes can be found at `http://dcg.epfl.ch/page-117408-en.html`.