

Geometric Graph Theory

Exercise session 6, March 25, 2015

Solutions of selected problems are accepted until April 1, 2015.

1. Show that if a graph has two vertex-disjoint odd cycles, then it cannot be drawn as a thrackle.
2. Let S be a set of n points in the plane such that the distance of every two points in S is at most 1. Color each point of S red, green or blue so that the distance between every two points of the same color is strictly smaller than 1.
3. Let S be a set of n points in the plane in general position. Prove that a shortest polygonal closed curve visiting all points from S is noncrossing. (A *polygonal closed curve* is a closed curve composed of straight-line segments, and is allowed to cross itself).
4. Prove that there is a constant $c > 0$ such that the following holds. Let k, n be positive integers. Then every convex geometric graph with n vertices and at most k pairwise disjoint edges has at most ckn edges.
5. Let G be a graph with m edges. Prove that G has a bipartite subgraph with at least $m/2$ edges as follows. Color each vertex of G independently at random red or blue, each color has probability $1/2$. Show that the expected number of edges having one endpoint red and the other endpoint blue is $m/2$.
-  6. Prove that every cycle of length at least 5 can be drawn as a thrackle.
- § 7. Let D be a plane drawing of a bipartite graph. Show that you can draw a simple curve that avoids the vertices and crosses every edge of D exactly once.

 — an optional homework. You may submit a written solution of this problem until the beginning of the next lecture and receive our feedback.

§ — an optional contest problem. You may submit a solution until the beginning of the next lecture. We encourage you to participate.