Covered Material

The references are according to the **Third Edition** of the book.

Lecture 1

Definition of $R(k)$, Proof of $R(3) = 6$, Proof of $R(k) \leq \binom{2k-2}{k-1}$, Construction for $R(k) > (k-1)^2$, Proposition 1.1.1.

Lecture 2

Definitions of tournament and Hamiltonian path, Theorem 2.1.1, Explicit construction for the lower bound on the number of Hamiltonian paths, Definition of $m(n)$, Proposition 1.3.1, Theorem 1.3.2.

Lecture 3

Definition of domination number of a graph, Theorem 1.2.2, Theorem 1.4.1.

Lecture 4

Beck's recoloring method. (Covered from *Ten Lectures on the Probabilistic Method, Joel Spencer*)

Lecture 5

Definitions and properties of the variance and covariance (Section 4.1), Chebyshev's Inequality (Theorem 4.1.1), Theorem 4.2.1, Theorem 4.6.1 (refer to Section 4.6).

Lecture 6

Lovász local lemma, Proposition 5.3.1.

Lecture 7

Application of conditional expectation: Erdős-Selfridge theorem, Theorem 5.4.1 in $\mathbb{R}^2$.

Lecture 8

Markov's inequality, Proof of Chebyshev's inequality using Markov's inequality, Theorem 4.3.1, Corollaries $4.3.2, 4.3.4, 4.3.5$, The following version of Theorem 4.4.1: The property $\omega(G) \geq 3$ has threshold function n^{-1}.

Lecture 9

Review of tools from the second moment method: Theorem 4.3.1, Corollaries 4.3.2, 4.3.3, 4.3.4, 4.3.5, Theorem 4.4.1, Definition of balanced graphs and examples (Section 4.4: Definition 1), Theorem 4.4.2.

Lecture 10

Review of Theorem 4.4.2, Theorem: If H is a graph and H' is a subgraph of H with maximum density, then the function $n^{-1/\rho(H')}$ is a threshold function for the event that H is a subgraph of $G(n,p)$. Theorem 4.4.3, Definition of monotone property for graphs (a property which is preserved under adding edges), Theorem (without proof): Every non-trivial monotone property has a threshold function. Theorem: The function $\log n/n$ is a threshold function for the property that $G(n,p)$ is connected.

Lecture 11

Section 4.5: Theorem 4.5.1, Corollary 4.5.2. Theorem 1 of page 41 (The probabilistic lens: High girth and high chromatic number).

Lecture 12

Theorem 8.1.1, Application: Proof of the probability that $G(n, c/n)$ is triangle-free reaches $e^{-c^3/6}$ as $n \to \infty$.

Lecture 13

Definition of discrepancy, Statement of the Van der Waerden's theorem: For any given positive integer k, there is some number N such that in any 2-coloring of the set $\{1, 2, \ldots, N\}$, there is a monochromatic arithmetic progression of length k. Statement of the Szemerédi's Theorem: For any positive integer k and positive real number ϵ, there exists a threshold number $N(k, \epsilon)$ such that for $N \geq N(k, \epsilon)$, every subset of $\{1, 2, \ldots, N\}$ with cardinality larger than ϵN contains an arithmetic progression of length k. Chernoff's inequality (Lemma 16.1 of the file "session 13" on the webpage), Theorem 16.2 of the same file.

Lecture 14

Review of the selected problems from the Sample Exam and exercise sessions.