Combinatorial Optimization – Problem Set 3

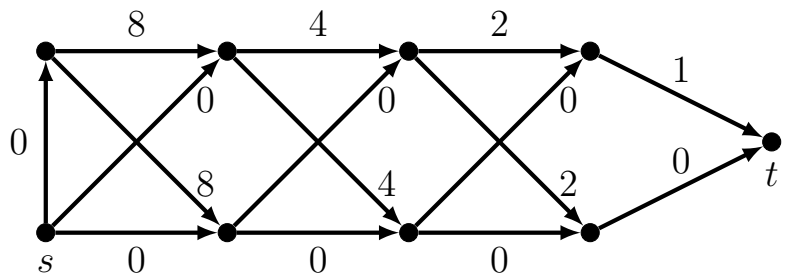
You can hand in one of the following problems at the start of Tuesday's problem session. Please explain your solution carefully. Don't forget to put your name.

Shortest Paths

1. Given a weighted directed graph G that may have negative edges but no negative cycle, and given a potential on G, show that a shortest ab-path can be found by changing G in such a way that Dijkstra's algorithm works on it.

2. Given a sequence $a_1, \ldots, a_n$ of real numbers, we want to find a consecutive subsequence with minimal sum, i.e. $i \leq j$ such that $\sum_{k=i}^{j-1} a_k$ is minimal (the sum $= 0$ when $i = j$). Show that such a sequence can be found using a quick version of Ford's algorithm, where edges are corrected in an order such that each one only needs to be corrected once.
   Carry this out for the sequence $-1, 3, -2, 1, -1, -1, 5, -2, 3, -2$.

3. Consider the following version of Ford's algorithm, which is simpler than the one in the notes: Set $d(a) = 0$, $d(v) = \infty$ for $v \neq a$; then repeatedly pick any edge $uv \in E(G)$ such that $d(v) > d(u) + w(uv)$, set $d(v) = d(u) + w(uv)$; stop when there are no such edges left.
   Use the graph below, and generalizations of it, to deduce that its running time is not polynomial.



4. Show that if you had a polynomial algorithm that finds a shortest path in any weighted directed graph (allowing negative cycles), then it would give you a polynomial algorithm to determine if any unweighted directed graph has a Hamilton cycle.