

# DCG - 30 years

## A New Era of Discrete & Computational Geometry

June 26 - July 1, 2016  
Monte Verità, Ascona

 Congressi  
Stefano Franscini

 ÉCOLE POLYTECHNIQUE  
FÉDÉRALE DE LAUSANNE

 ETH zürich

 Springer

abstracts

## Last minute information

- It would greatly simplify the administrative procedures if, before coming to Monte Verità, all participants would register (for the second time!) and prepay their bills using a credit card on the link

[www.bi.id.ethz.ch/eventsOnline/anonymous/registerParticipantForConference.faces?webboname=Conference&loadid=3l3o75s-d0b0ix-hhnaftz-1-ie2f4sva-8ec9](http://www.bi.id.ethz.ch/eventsOnline/anonymous/registerParticipantForConference.faces?webboname=Conference&loadid=3l3o75s-d0b0ix-hhnaftz-1-ie2f4sva-8ec9)

The amounts to be paid are sent to each participant by email. We apologize for the inconvenience.

- The check-in in Hotel Monte Verità starts at 3 pm. If you arrive much earlier or late in the evening, please inform the organizers beforehand to try to make special arrangements.
- On Sunday evening from 6 to 7pm, there will be a welcome drink at the Centro.
- On Sunday and Wednesday evenings, no dinner will be served: a list of nearby restaurants can be found on page 17 of this booklet.
- On Thursday night, there will be a conference dinner, which is free for all participants. Tickets for accompanying persons can be purchased for 35 CHF (less than half of the price) at the conference site.
- The Ascona Jazz Festival is running from June 23 to July 2. For more details, consult <http://www.jazzascona.ch>.

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## Venue

**Congressi Stefano Franscini**

**Monte Verità**

**Via Collina 84**

**CH-6612 Ascona**

**More information online at:**

<http://www.csf.ethz.ch/how-to-find-us.html>



# How to reach the conference center

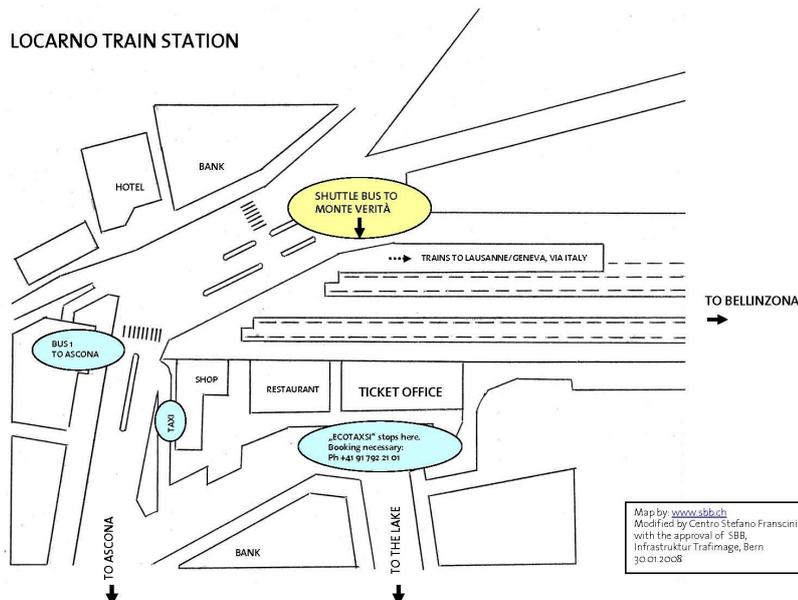
## Reach Locarno

In order to reach the Congressi Stefano Franscini at Monte Verità in Ascona, whether you fly or travel by train from within Europe, you must first get to Locarno, the closest railway station to Ascona.

## From Locarno to Monte Verità

Once in Locarno, you can reach Monte Verità as follows

- A free 11-seater shuttle bus to Monte Verità will be available on the day of arrival, June 26. The departure times of the shuttle are 1:00 pm, 1:40 pm, 2:20 pm, 3:00 pm, 3:40 pm, 4:20 pm. The shuttle meeting point is on the right side of the train platforms in Locarno (see map at the end of this section).
- Public bus: Bus Nr. 1 from Locarno railway station to Ascona (stop Ascona Posta) and then Bus Nr. 5 ("Buxi") from Ascona to Monte Verità. Duration: 15-20 minutes for Line Nr 1. + 8 minutes for Line Nr. 5 Cost: 2.20 CHF for one way ticket from Locarno to Ascona + 2.00 CHF for a one way ticket of the Buxi from Ascona to Monte Verità Schedules Line Nr. 1 and Line Nr. 5; please note: the bus line Nr. 1 has different schedule for Monday-Saturday (Lunedì-sabato e feriali) and for Sunday and holidays (Domenica e festivi) the line Nr. 5 has a rather low frequency, so make sure you check the schedule. If the schedule of Bus Nr. 5 is not convenient for your arrival time, you can reach Monte Verità from Ascona Posta by walking approx. 25-30 minutes uphill along the road named Via Collina
- Taxi: There are several taxis available at Locarno railway station. The cheapest rates are usually with EcoTaxi Booking necessary: phone +41 91 792 21 01 or 0800 321 321 (free number only from Swiss phones, or via Skype) Eco Taxi from Locarno: Locarno - Monte Verità Duration: approx. 15 min. Cost: approx. CHF 24.-



The field of "Discrete & Computational Geometry" was created thirty years ago, by joining the classical mathematical field of Discrete Geometry with the emerging computer science discipline of Computational Geometry. The marriage has proved to be fruitful: A new generation of researchers treats deep problems from one side, with insights, tools and theory that arose on the other side - with great successes. This meeting will bring together key researchers and promising young colleagues to assess the state of the art and discuss further steps and directions.

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## Organizers



**Pankaj Agarwal**



**Ken Clarkson**



**Herbert Edelsbrunner**



**János Pach**



**Emo Welzl**



**Günter M. Ziegler**

## Speakers

Karim Adiprasito

Jin Akiyama

Imre Bárány

Alexander Barvinok

Friedrich Eisenbrand

Jacob Fox

Larry Guth

Sariel Har-Peled

Piotr Indyk

Gil Kalai

Roman Karasev

Anna Lubiw

Rom Pinchasi

Günter Rote

Micha Sharir

Andrew H. Suk

Subhash Suri

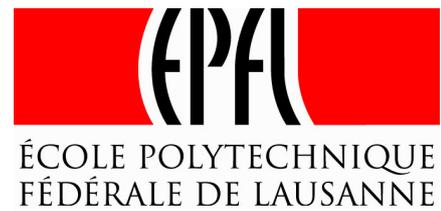
Gábor Tardos

Csaba Tóth

Pavel Valtr

Maryna Viazovska

## Sponsors



# Participants

Eyal Ackerman	Michael Hoffmann	Vera Rosta
Karim Adiprasito	Piotr Indyk	Günter Rote
Pankaj Agarwal	Grigory Ivanov	Natan Rubin
Oswin Aichholzer	Gil Kalai	Raman Sanyal
Jin Akiyama	Roman Karasev	Micha Sharir
Alexandr Andoni	Gyula Károlyi	Shakhar Smorodinsky
Boris Aronov	Michael Kerber	Noam Solomon
Imre Bárány	Andrei Kupavskii	Jozsef Solymosi
Alexander Barvinok	Jeffrey Lagarias	Marc Strauss
Matthias Beck	Nati Linial	Andrew H. Suk
Mark De Berg	Jesus De Loera	Subhash Suri
Otfried Cheong	Anna Lubiw	Konrad Swanepoel
Ken Clarkson	Leonardo Martínez-Sandoval	László Székely
Éva Czabarka	Viola Mészáros	Enikő Szép
Tamal Dey	Malte Milatz	Martin Tancer
Thao Do	Joseph Mitchell	Gábor Tardos
Herbert Edelsbrunner	Hossein Mojarad	Takeshi Tokuyama
Friedrich Eisenbrand	Luis Montejano	Csaba Tóth
Esther Ezra	Nabil Mustafa	Jorge Urrutia
Brittany Fasy	Márton Naszódi	Claudiu Valculescu
Stefan Felsner	János Pach	Pavel Valtr
Jacob Fox	Thang Pham	Maryna Viazovska
Komei Fukuda	Jeff Phillips	Uli Wagner
Zoltan Füredi	Alexander Pilz	Hong Wang
Bernd Gärtner	Rom Pinchasi	Yusu Wang
Andrei Gavriluk	Richard Pollack	Emo Welzl
Xavier Goaoc	Sharath Raghvendra	Manuel Wettstein
Larry Guth	Benjamin Raichel	Ben Yang
Dan Halperin	Andrei Raigorodskii	Joshua Zahl
Sariel Har-Peled	Orit Raz	Frank de Zeeuw
Julia Hockenmaier	Jürgen Richter-Gebert	Günter M. Ziegler

# Schedule

Tentative Schedule	Monday (June 27)	Tuesday (June 28)	Wednesday (June 29)	Thursday (June 30)	Friday (July 1)
9:15-10:00	Kalai	Fox	Indyk	Sharir	Tardos
10:15-11:00	Viazovska	Guth	Karasev	Lubiw	Tóth
11:00-11:30	Coffee	Coffee	Coffee	Coffee	Coffee
11:30-12:15	Adiprasito	Har-Peled	Suk	Pinchasi	Valtr
12:30-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00-16:00	Work	DCG meeting & work	Optional excursion, sightseeing, etc.	Work	Departure
16:00-16:15	Bárány	Young Scientists Session (see below)		Rote	
16:15-16:45					
16:45-17:10	Coffee			Suri	
17:10-17:55	Barvinok				
18:00-18:45	Eisenbrand	Springer/DCG reception		Conference Dinner	
19:00-20:15	Dinner	Dinner			
20:30-21:30		Public lecture of Akiyama			

## Young Scientists Session on Tuesday

16:00-16:15	Andrey Kupavskii
16:20-16:35	Leonardo Martinez
16:40-17:55	Benjamin Raichel
17:00-17:15	Orit Raz
17:20-17:35	Manuel Wettstein
17:40-17:55	Ben Yang

**There is a 500 CHF Award for the Best Presentation by a Young Scientist.**

# List of abstracts

(in alphabetical order of the speakers)

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## Hodge theory for spheres and matroids

**Speaker:** Karim Adiprasito

**Abstract:** We discuss applications of Hodge theory, a part of algebraic geometry, to problems in combinatorics. We moreover discuss situations in which these deep algebraic theorems themselves can be shown combinatorially, extending our knowledge of cohomology of closed currents in toric varieties.



## Enter the World of Math and Art

**Speaker:** Jin Akiyama

**Abstract:** This lecture aims to bring out the best of both worlds: Mathematics and Arts! Relying on the following theorems on nets of polyhedra, the discussion will unleash the hidden artist in every participant. Together we will create beautiful pictures, comparable to the masterpieces of M. C. Escher.



*Terminology and Definition:* An isotetrahedron is a tetrahedron with all faces congruent. Conway criterion is stated as follow: A given figure can tile the plane using only translations and 180 degree rotations if its perimeter can be divided into six parts by six consecutive points  $A, B, C, D, E,$  and  $F$  (all located on its perimeter) such that: 1. The perimeter part  $AB$  is congruent by translation  $\tau$  to the perimeter part  $ED$  in which  $\tau(A) = E, \tau(B) = D$ : i.e.,  $AB \parallel ED$ ; 2. Each of the perimeter parts  $BC, CD, EF,$  and  $FA$  is centrosymmetric, that is, each of them coincides with itself when the figure is rotated by 180 degrees around its midpoint; 3. Some of the six points may coincide but at least three of them must be distinct.

- *Theorem A.* Every net of an isotetrahedron tiles the plane. Moreover, every net of an isotetrahedron satisfies the Conway criterion.

- *Theorem B.* Let  $P$  be a polyhedron, and let  $D_1, D_2$  be non-crossing dissection trees of  $P$ . Then  $N_1$  is reversible to  $N_2$ , where  $N_1, N_2$  is a net obtained by dissecting its surface along edges of  $D_1, D_2$ , respectively.

- *Theorem C.* Let  $D_1$  be an arbitrary dissection tree of an isotetrahedron  $T$ . Then there exists a dissection tree  $D_2$  of  $T$ , which doesn't cross  $D_1$ . A pair of nets  $N_i$  obtained by dissecting along  $D_i$  ( $i = 1, 2$ ) is reversible each other, and each  $N_i$  tiles the plane.

- *Theorem D.* There are exactly 23 tessellation polyhedra with regular polygonal faces. Moreover, for each tessellation polyhedron with regular polygonal faces, there is at least one e-net which is a Conway tile.

- *Theorem 1.* For any Conway tile  $P$ , there exists a Conway tile  $Q$  such that  $P$  is reversible to  $Q$ .

- *Theorem 2.* An e-net of some tessellation polyhedron with regular polygonal faces can be refolded into an isotetrahedron.

## Convex cones, integral zonotopes, and their limit shape

Speaker: Imre Bárány

**Abstract:** Given a pointed cone  $C$  in  $\mathbb{R}^d$ , an integral zonotope in  $C$  is the Minkowski sum of segments of the form  $[0, z_i]$ , ( $i = 1, \dots, m$ ) where  $z_i$  is an integer vector from  $C$ . The endpoint of this zonotope is the sum of the  $z_i$ . The collection  $T(C, k)$  of integral polytopes in  $C$  with endpoint  $k$  is a finite set. We show that the zonotopes in  $T(C, k)$  have a limit shape as  $k$  goes to infinity. We also establish the order of magnitude of the face numbers of a typical zonotope in  $T(C, k)$ . This is joint work with Julien Bureaux and Ben Lund.




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## Complex geometry and computational complexity

Speaker: Alexander Barvinok

**Abstract:** I'll present a computationally efficient way to approximate a multivariate polynomial in a complex domain provided the polynomial does not have zeros in a slightly larger domain. Examples include the permanent of a real or complex matrix, its multi-dimensional versions, the independence polynomial of a graph and other partition functions.




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## Max-sum diversity via convex programming

Speaker: Friedrich Eisenbrand

**Abstract:** Diversity maximization is an important concept in information retrieval, computational geometry and operations research. Usually, it is a variant of the following problem: Given a ground set, constraints, and a function  $f(\cdot)$  that measures diversity of a subset, the task is to select a feasible subset  $S$  such that  $f(S)$  is maximized. The *sum-dispersion* function  $f(S) = \sum_{x, y \in S} d(x, y)$ , which is the sum of the pairwise distances in  $S$ , is in this context a prominent diversification measure. The corresponding diversity maximization is the *max-sum* or *sum-sum diversification*.

Many recent results deal with the design of constant-factor approximation algorithms of diversification problems involving sum-dispersion function under a matroid constraint. In this paper, we present a PTAS for the max-sum diversification problem under a matroid constraint for distances  $d(\cdot, \cdot)$  of *negative type*. Distances of negative type are, for example, metric distances stemming from the  $\ell_2$  and  $\ell_1$  norm, as well as the cosine or spherical, or Jaccard distance which are popular similarity metrics in web and image search.

Joint work with Alfonso Cevallos and Rico Zenklusen



## Tools in Discrete Geometry

**Speaker:** Jacob Fox

**Abstract:** It is an exciting time for discrete and computational geometry, with substantial progress on a variety of major problems have come from using tools from algebra, analysis, combinatorics, geometry, number theory, probability, and topology, or some combination. I will discuss several prominent examples, and how we can learn from them.



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## Ruled surfaces in incidence geometry

**Speaker:** Larry Guth

**Abstract:** We will survey the role of ruled surface theory in incidence geometry. In algebraic geometry, a ruled surface is an algebraic variety which contains a line through every point. In recent years, ruled surface theory has played a role in several proofs in incidence geometry, starting in the work that Nets Katz and I did on the distinct distance problem in the plane. We will explain how the connection between combinatorics and ruled surface theory works, and then discuss some open problems and directions.



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## Beyond planarity: On geometric intersection graphs

**Speaker:** Sarel Har-Peled

**Abstract:** Many efficient algorithms had been developed over the years for planar graphs and more general graphs such as low genus graphs. Intersection graphs of geometric objects (in low dimensions) with some additional properties, such as fatness or low density, provide yet another family of graphs for which one can design better algorithms.

This family is a vast extension of planar graphs, and yet is still algorithmically tractable for many problems. In this talk, we will survey this class of graphs, and some algorithms and intractability results known for such graphs, and outline open problems for further research.



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## New Algorithms for Similarity Search in High Dimensions

**Speaker:** Piotr Indyk

**Abstract:** Similarity search is a fundamental computational task that involves searching for similar items in a large collection of objects. This task is often formulated as the nearest neighbor problem: given a database of  $n$  points in a  $d$ -dimensional space, devise a data structure that, given any query point, quickly finds its nearest neighbor in the database.



The problem has a large number of applications in a variety of fields, such as machine learning, databases, natural language processing and computer vision. Many of those applications involve data sets that are very high dimensional. Unfortunately, all known algorithms for this problem require query time or data structure size that are exponential in the dimension, which makes them inefficient when the dimension is high enough. As a result, over the last two decades there has been a considerable effort focused on developing approximate algorithms that can overcome this "curse of dimensionality"

A popular framework for designing such algorithms is Locality Sensitive Hashing (LSH). It relies on the existence of efficiently computable random mappings (LSH functions) with the property that the probability of collision between two points is related to the distance between them. The framework is applicable to a wide range of distances and similarity functions, including the Euclidean distance. For the latter metric, it is known that the "basic" application of the LSH function yields e.g., a 2-approximate algorithm with a query time of roughly  $dn^{(1/4)}$ , for a set of  $n$  points in a  $d$ -dimensional space.

In this talk I will describe recent data structures that offer significant improvements over the aforementioned bounds. The improvement is achieved by performing \*data-dependent\* hashing, which constructs the random hash functions in a way that depends on the data distribution. I will also describe a recent implementation of the "core" component in the aforementioned algorithms, which empirically outperforms widely used variants of LSH. The implementation is available at <http://falconn-lib.org/>

This talk covers joint work with Alexandr Andoni, Thijs Laarhoven, Huy Nguyen, Ilya Razenshteyn and Ludwig Schmidt.

## Combinatorics and Geometry of subsets of the discrete cube

Speaker: Gil Kalai

**Abstract:** We will start with the notions of influence and noise sensitivity for subsets of the discrete cube described by their characteristic functions. The influence of a variable (or a set of variables) on a function is the probability that changing the value of the variable(s) can change the value of the function. The noise-sensitivity of a function is the probability that for a random assignment to the variables adding a random independent noise will change the value of the function.



We will look at some old and new results and open problems and mention connections with Harmonic analysis, sharp threshold phenomena, percolation, random graphs, extremal combinatorics, correlation inequalities, voting, and computation. New results that I will present are based on joint works with Jeff Kahn, Jean Bourgain, Ehud Friedgut, Guy Kindler, Nathan Keller, and Elchanan Mossel.

## Symplectic ideas in convex geometry

Speaker: Roman Karasev

**Abstract:** I am going to speak about situations where the symplectic point of view allows to better understand some problems in convex geometry. Sometimes it allows to prove something in convex geometry, sometimes it only provides a useful intuition about what statement to prove in convex geometry.



In particular, I am going to discuss Mahler's conjecture and the related problem of understanding what is a symplectic ball.

## Flipping Edge-Labelled Triangulations

Speaker: Anna Lubiw

**Abstract:** In a triangulation of points in the plane, a flip replaces one edge by the opposite edge of its surrounding quadrilateral when that quadrilateral is convex. A foundational result of Lawson's is that any triangulation can be transformed to any other via a sequence of flips. Although flips have been extensively studied, almost nothing is known about where individual edges move during the course of a sequence of flips.



We formalize this by attaching labels to the edges of the triangulation, with the convention that when edge  $e$  flips to edge  $e'$ , the label of  $e$  transfers to  $e'$ . The problem we explore is: Given two edge-labelled triangulations of a point set in the plane, can one be transformed to the other using flips? A necessary condition is that for every label, the initial and final edges with that label, say  $e$  and  $f$ , lie in the same "orbit", i.e. there is some sequence of flips in some sequence of triangulations that takes  $e$  to  $f$ .

This talk will be about several joint results. With Prosenjit Bose, Vinayak Pathak, and Sander Verdonschot we formulated the Orbit Conjecture—that the above necessary condition is sufficient. We proved the conjecture for convex point sets and for spiral polygons, providing tight bounds on the number of flips required. With Jim Geelen and Vinayak Pathak we proved the analogue of the Orbit Conjecture for matroids. Finally, in work in progress with Zuzana Masarova and Uli Wagner, we have proved the Orbit Conjecture using some topological results on the dual of the simplicial complex whose maximal simplices are the triangulations.

## The maximum perimeter of the convex hull of $n$ non-separable unit discs.

Speaker: Rom Pinchasi

**Abstract:** Let  $F$  be a family of  $n$  unit discs in the plane with the property that every line either crosses one of the discs or it determines a half-plane that contains all the discs. We show that the perimeter of the convex hull of the union of all discs in  $F$  is at most  $2\pi + 4(n - 1)$ . This bound is tight. This is a joint work with Uri Rabinovich.



## Congruence Testing of Point Sets in Three and Four Dimensions

Speaker: Günter Rote

**Abstract:** I will survey algorithms for testing whether two point sets are congruent, that is, equal up to an Euclidean isometry. I will introduce the important techniques for congruence testing, namely dimension reduction and pruning, or more generally, condensation. I will illustrate these techniques on the three-dimensional version of the problem, and indicate how they lead for the first time to an algorithm for four dimensions with near-linear running time (joint work with Heuna Kim).



On the way, we will encounter some beautiful and symmetric mathematical structures, like the regular polytopes, and Hopf fibrations of the three-dimensional sphere in four dimensions.

## Eliminating cycles, cutting lenses, and bounding incidences

Speaker: Micha Sharir

**Abstract:** The talk covers two unrelated topics in combinatorial geometry that have recently reached a confluence:

incidences between points and curves in the plane, or surfaces in higher dimensions, and elimination of cycles in the depth relation of lines in 3-space. Recent progress on the latter problem, inspired by the new algebraic machinery of Guth and Katz, has yielded a nearly tight bound, of roughly  $n^{3/2}$ , on the number of cuts needed to eliminate all cycles for a set of  $n$  lines, or simply-shaped algebraic curves, in 3-space.

This in turn leads to a similar bound on the number of cuts that are needed to turn a collection of  $n$  constant-degree algebraic arcs in the plane into a collection of pseudo-segments (i.e., each pair of the new subarcs intersect at most once). This leads, among several other applications, to improved incidence bounds between points and algebraic arcs in the plane, which are better than the older general bound of Pach and Sharir, for any number of "degrees of freedom" of the curves. It also leads to several new bounds for incidences between points and surfaces in three dimensions.

Based on joint works with Boris Aronov, Noam Solomon, and Joshua Zahl.




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## On the Erdős-Szekeres convex polygon problem

Speaker: Andrew H. Suk

**Abstract:** Let  $ES(n)$  be the smallest integer such that any set of  $ES(n)$  points in the plane in general position contains  $n$  points in convex position. In their seminal 1935 paper, Erdős and Szekeres showed that  $ES(n) \leq \binom{2n-4}{n-2} + 1 = 4^{n-o(n)}$ . In 1960, they showed that  $ES(n) \geq 2^{n-2} + 1$  and conjectured this to be optimal. Despite the efforts of many researchers, no improvement in the order of magnitude has been made on the upper bound over the last 81 years. In this talk, we will sketch a proof showing that  $ES(n) = 2^{n+o(n)}$ . We will also discuss several related open problems including a higher dimensional variant, and on mutually avoiding sets.




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## Simple Problems Made Hard: CG Meets Data Uncertainty

Speaker: Subhash Suri

**Abstract:** We consider two classical problems of computational geometry—convex hull membership and hyperplane separability—under some simple models of data uncertainty. The convex hull membership problem asks: given  $n$  uncertain points, what is the probability that another point  $q$  lies inside their convex hull? The hyperplane separability problem asks: what is the probability that two uncertain point sets are linearly separable? We show the following results:

1. The convex hull membership for  $n$  points in  $d$  dimensions can be solved in  $O(n^{d-1})$  time.
2. The hyperplane separability of  $n$  points in  $d$  dimensions can be solved in  $O(n^d)$  time.



3. The hyperplane separability in  $d$  dimension is equivalent to convex hull membership in dimension  $d + 1$ .
  4. The  $d$ -dimensional separability is  $(d + 1)$ -SUM hard.
- Time permitting, we will also discuss some range query problems under these models.

## Coloring geometric intersection and disjointness graphs

Speaker: Gábor Tardos

**Abstract:** The clique number  $\omega(G)$  and chromatic number  $\chi(G)$  of any graph  $G$  satisfy the trivial inequality  $\omega(G) \leq \chi(G)$ . Graphs where equality holds for every induced subgraph are called perfect and are well studied. Gyárfás and Lehel introduced a notion of quasi-perfectness, namely they call a graph family  $\chi$ -bounded if there exists a function  $f$  such that  $\chi(G) \leq f(\omega(G))$  holds for every graph  $G$  in the family. We study the *chi*-boundedness of intersection and disjointness graphs of various geometric objects. We show for example that intervals in 3D have  $\chi$ -bounded disjointness graphs but simplexes do not. Joint work with János Pach and Géza Tóth.



## Weakly simply polygons: recognition and reconstruction

Speaker: Csaba Tóth

**Abstract:** A polygon is weakly simple if it can be perturbed into a simple polygon by moving each vertex within an arbitrarily small disk. Weakly simple polygons arise as shortest curves in a given homotopy class: they have no self-crossings but may have overlapping edges. Geometric algorithms designed for simple polygons often work for weakly simple polygons, as well, if a (symbolic) perturbation is given. We consider two algorithmic problems that are easy for simple polygons but become challenging for weakly simple polygons.

Given a polygon with  $n$  vertices in the plane, we can decide whether it is weakly simple in  $O(n \log n)$ -time, improving upon the previous runtime bound  $O(n^2)$  by Chang, Erickson, and Xu (2015). We show that deciding whether a given set of line segments in the plane form the edge set of a weakly simple polygon is NP-complete; and we have bounds on the cost of augmenting a set of pairwise noncrossing line segments into a weakly simple polygon. (Joint work with Hugo Akitaya, Greg Aloupis, and Jeff Erickson.)



## On different measures of non-convexity

Speaker: Pavel Valtr

Different measures of non-convexity of a set in  $R^d$  will be discussed. One set of considered problems is unified by the following definitions. The invisibility graph  $I(X)$  of a set  $X \subseteq R^d$  is a (possibly infinite) graph whose vertices are the points of  $X$  and two vertices are connected by an edge



if and only if the straight-line segment connecting the two corresponding points is not fully contained in  $X$ . We consider the following three parameters of a set  $X$ : the clique number  $\omega(I(X))$ , the chromatic number  $\chi(I(X))$  and the minimum number  $\gamma(X)$  of convex subsets of  $X$  that cover  $X$ . Observe that  $\omega(X) \leq \chi(X) \leq \gamma(X)$  for any set  $X$ , where  $\omega(X) := \omega(I(X))$  and  $\chi(X) = \chi(I(X))$ .

Here is a sample of results comparing the above three parameters:

1. Let  $X \subseteq \mathbb{R}^2$  be a planar set with  $\omega(X) = \omega < \infty$  and with no one-point holes. Then  $\gamma(X) \leq O(\omega^4)$ .
2. For every planar set  $X$ ,  $\gamma(X)$  can be bounded in terms of  $\chi(X)$ .
3. There are sets  $X$  in  $\mathbb{R}^5$  with  $\chi(X) = 3$ , but  $\gamma(X)$  arbitrarily large.

Another set of problems is related to the following definitions. Let  $S$  be a subset of  $\mathbb{R}^d$  with finite positive Lebesgue measure. The *Beer index of convexity*  $b(S)$  of  $S$  is the probability that two points of  $S$  chosen uniformly independently at random see each other in  $S$ . The *convexity ratio*  $c(S)$  of  $S$  is the Lebesgue measure of the largest convex subset of  $S$  divided by the Lebesgue measure of  $S$ .

Here is a planar result comparing the above parameters: Every set  $S \subseteq \mathbb{R}^2$  with simply connected components satisfies  $b(S) \leq \alpha \cdot c(S)$  for an absolute constant  $\alpha$ , provided  $b(S)$  is defined. Related higher-dimensional results and questions will be also discussed.

The talk will review results of M. Balko, S. Cabello, J. Cibulka, V. Jelínek, M. Korbelář, M. Kynčl, J. Matoušek, V. Mészáros, G. Rote, M. Saumell, R. Stolař, P. Valtr, B. Walczak, and many others.

## The sphere packing problem in dimensions 8 and 24

Speaker: Maryna Viazovska

**Abstract:** In this talk we will present a solution of the sphere packing problem in dimensions 8 and 24 using modular forms. In 2003 N. Elkies and H. Cohn proved that the existence of a real function satisfying certain constraints leads to an upper bound for the sphere packing constant. Using this method they obtained almost sharp estimates in dimensions 8 and 24. We will show that functions providing exact bounds can be constructed explicitly as certain integral transforms of modular forms. Therefore, the sphere packing problem in dimensions 8 and 24 is solved by a linear programming method.



## Young scientists

(in alphabetical order)

### New Lower Bounds for $\epsilon$ -nets

Speaker: Andrey Kupavskii

**Abstract:** Following the seminal work by Haussler and Welzl, the use of small  $\epsilon$ -nets has become a standard technique for solving algorithmic and extremal problems in geometry and learning theory. Two significant recent developments are: (i) an upper bound on the size of the smallest  $\epsilon$ -nets for set systems, as a function of their so-called shallow-cell complexity, due to Chan, Grant, Köneemann, and Sharpe; and (ii) the construction of a set system whose members can be obtained by intersecting a point set in  $\mathbb{R}^4$  by a family of halfspaces such that the size of any  $\epsilon$ -net for them is  $\Omega(\frac{1}{\epsilon} \log \frac{1}{\epsilon})$ , due to Pach and Tardos. In this talk we complete both of these avenues of research. We (i) give a lower bound,



matching the result of Chan *et al.*, and (ii) generalize the construction of Pach and Tardos to half-spaces in  $\mathbb{R}^d$ , for any  $d \geq 4$ , to show that the general upper bound,  $O(\frac{d}{\epsilon} \log \frac{1}{\epsilon})$ , of Haussler and Welzl for the size of the smallest  $\epsilon$ -nets is tight.

## A Hall-type theorem for points in general position

**Speaker: Leonardo Martinez-Sandoval**

**Abstract:** Hall's theorem gives conditions under which a family of sets has a system of distinct representatives. In this work we provide a geometric generalization.

For a family  $F = \{X_1, X_2, \dots, X_m\}$  of finite sets in  $\mathbb{R}^d$  a system of general position representatives is a set  $\{x_1, x_2, \dots, x_m\}$  of points in  $\mathbb{R}^d$  in general position such that  $x_i \in X_i$ . For a finite set  $X$  of points in  $\mathbb{R}^d$ , we define  $\varphi(X)$  as the size of the largest affinely independent subset of  $X$ .

We show that there exists a function  $f_d$  such that if for each index subset  $I \subset [m]$  we have  $\varphi(\bigcup_{i \in I} X_i) \geq f_d(|I|)$ , then  $F$  has a system of general position representatives. We use topological techniques in the spirit of the celebrated Aharoni and Haxell's generalization of Hall's theorem. During the talk we will give a sketch of the proof and we will provide some open problems concerning the best possible order of  $f_d$ . This is joint work with Andreas Holmsen and Luis Montejano.



## Avoiding the Global Sort: A Faster Contour Tree Algorithm

**Speaker: Benjamin Raichel**

**Abstract:** Given a scalar field  $f$ , each connected component of a level set of  $f$  is called a contour. The *contour tree* is a fundamental topological structure that tracks the evolution of contours of  $f$ , and has numerous applications in data analysis and visualization, where for example a contour may represent a storm front in a barometric pressure map, or the boundary of a flame in a combustion simulation. All existing algorithms to compute contour trees begin with a global sort of at least all critical values of  $f$ , which can require (roughly)  $\Omega(n \log n)$  time.

We present the first algorithm whose time complexity depends on the contour tree structure, avoiding the global sort for many natural instances. For balanced trees our algorithm runs in  $O(n\alpha(n))$  time, and more generally we prove strong optimality properties of our algorithm, parameterized on the structure of the contour tree.



## Configurations of lines in 3-space and rigidity of planar structures

**Speaker: Orit Raz**

**Abstract:** Let  $L = (\ell_1, \dots, \ell_n)$  be a sequence of  $n$  lines in  $\mathbb{R}^3$ . We define the **intersection graph**  $G_L = ([n], E)$  of  $L$ , where  $[n] := \{1, \dots, n\}$ , and with  $(i, j) \in E$  if and only if  $i \neq j$  and the corresponding lines  $\ell_i$  and  $\ell_j$  intersect, or are parallel (or coincide). For a graph  $G = ([n], E)$ , we say that a sequence  $L$  realizes  $G$  if  $G \subset G_L$ . In the talk I will introduce a characterization of those graphs  $G$ , such that, every (generic) realization  $G$ , consists of lines that are either all concurrent or all coplanar.

The result is inspired by connections we have found between configurations of lines in 3-space and



the classical notion of graph rigidity of planar structures. The interaction goes in both directions: Properties established in the context of graph rigidity either can be interpreted directly as properties of line configurations in space, or lead the way to conjectures concerning the intersection patterns of lines in space. These general statements about lines in space, apart from their independent interest, can then be used back in the context of graph rigidity to get new information in this context. This will be illustrated in the talk.

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## Trapezoidal Diagrams, Upward Triangulations, and Prime Catalan Numbers

**Speaker: Manuel Wettstein**

**Abstract:** We introduce the notion of a trapezoidal diagram of a plane straight-line graph  $G$ . Loosely speaking, such a diagram encodes the vertical visibility relations between edges and vertices of  $G$ .

We study the number of such diagrams if the graphs  $G$  are either (a) perfect matchings or (b) triangulations with  $n$  vertices. We give bijections to (a) 3-dimensional balanced bracket expressions and (b) a specific subset of indecomposable such expressions.

While the former corresponds to a well-known 3-dimensional generalization of Catalan numbers, the latter gives rise to a previously unknown counting sequence which we call prime Catalan numbers. Exponential growth rates can then be determined to be  $a^n$  and  $b^n$ , respectively, where  $a = \sqrt{27} \approx 5.196$  and  $b = 27/(729 * \sqrt{3}/(40 * \pi) - 9)^3 \approx 23.459$ .

Incidentally, in case (b) these diagrams are closely related to upward triangulations, i.e., directed maximal planar graphs that allow a plane embedding in which all edges are drawn as  $y$ -monotone curves pointing upwards.

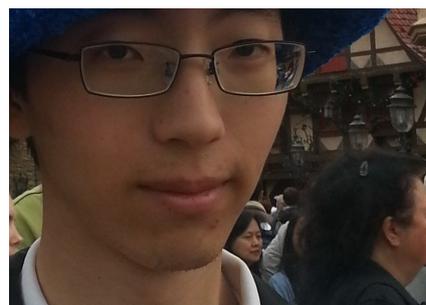
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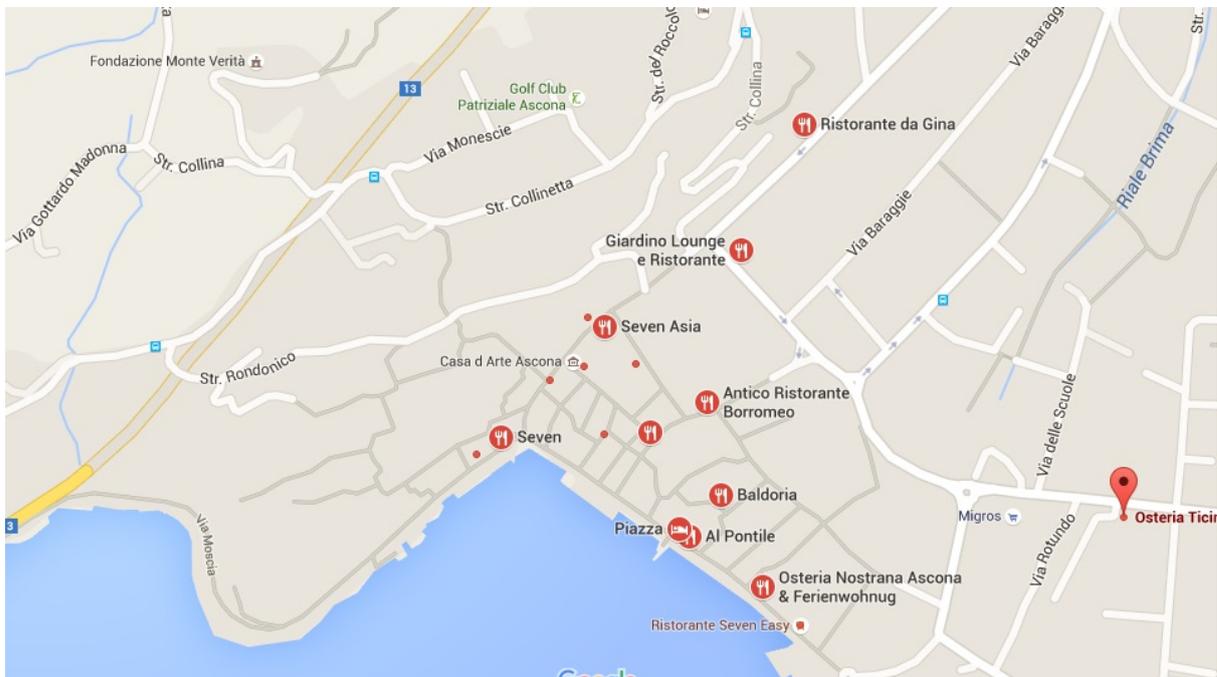
## Generalizations of Joints Problem

**Speaker: Ben Yang**

**Abstract:** In 2008 Guth and Katz proved the joints conjecture, showing that for any  $N$  lines in  $\mathbb{R}^3$ , there are at most  $O(N^{3/2})$  points at which three lines intersect non-coplanarly. We will generalize this result to sets of varieties of bounded degree in  $\mathbb{R}^n$  and prove almost sharp bound on the number of multijoints. In particular, we will prove that for any  $N$  2-planes in  $\mathbb{R}^6$ , there are at most  $O(N^{3/2+\epsilon})$  points at which three 2-planes intersect and span  $\mathbb{R}^6$ . The main tools are polynomial partitioning for varieties and induction on both dimensions and numbers of varieties.



# Where to eat



## Antico Ristorante Borromeo

Address: Via Collegio 16, 6612 Ascona  
 Telephone: +41 91 791 92 81  
 Website: <http://borromeoascona.blogspot.ch/>

## Osteria Nostrana

Address: Piazza Giuseppe Motta, 6612 Ascona  
 Telephone: +41 91 791 51 58  
 Website: <http://www.ristoranti-ff.ch/nostranaen.htm>

## Osteria Ticino (da Ketty and Tommy)

Address: Via Muraccio 20, 6612 Ascona  
 Telephone: +41 91 791 35 81  
 Website: <http://www.osteria-ticino.ch/restaurant.html>

## Osteria/Grotto Baldoria

Address: Via S. Ombono, Ascona, Switzerland  
 Telephone: +41 91 791 32 98  
 Website: <http://www.grottobaldoria.ch/wb/index.php>

## Ristorante da Gina

Address: Viale Monte Verità 19, 6612 Ascona  
 Telephone: +41 91 791 27 40  
 Website: <http://dagina.ch/>

## Seven Asia

Address: Ristorante Asia, Via Borgo 19, CH – 6612 Ascona  
 Telephone: +41 91 786 96 76  
 Website: <http://seven.ch/asia/de/angebot.html>

