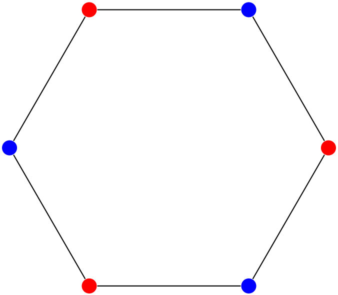
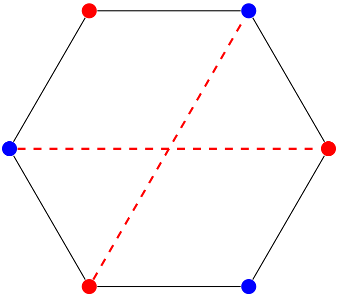
$K_{3,3}$ is not planar



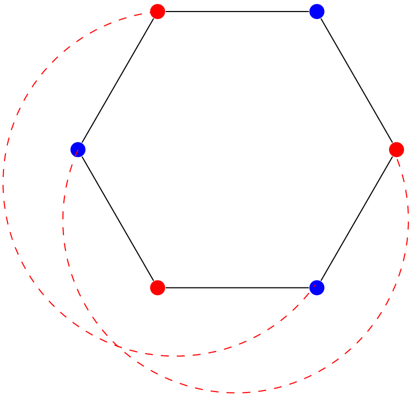
Take a 6-cycle in $K_{3,3}$

$K_{3,3}$ is not planar



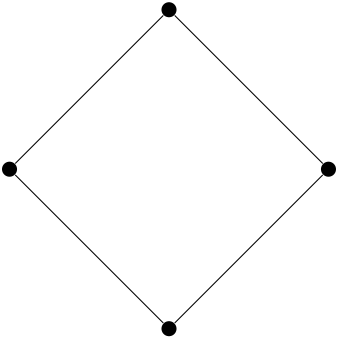
Out of three remaining edges two either go inside the cycle,

$K_{3,3}$ is not planar



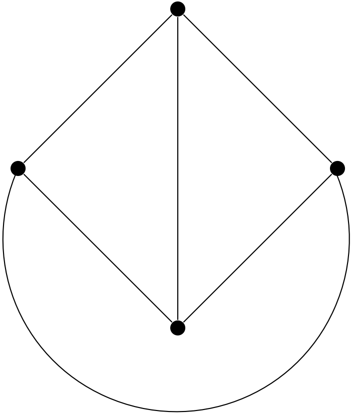
or two go outside the cycle.

K_5 is not planar



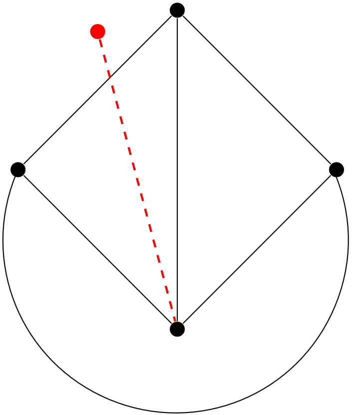
Take any 4-cycle in K_5.

K_5 is not planar



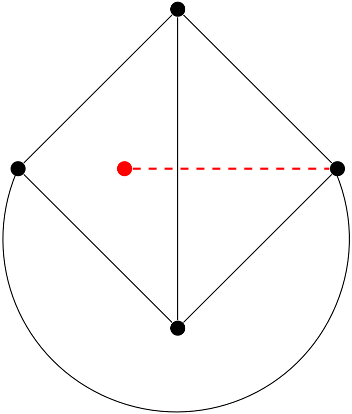
One of its two diagonals goes inside, the other goes outside.

K_5 is not planar



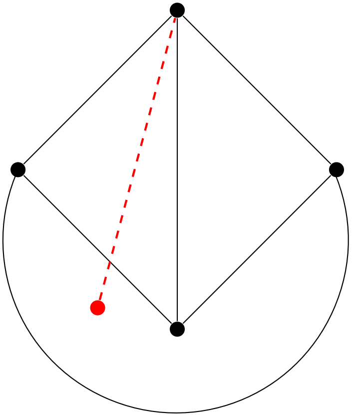
No matter where we put the fifth vertex, it cannot be connected to one of the vertices without crossings.

K_5 is not planar



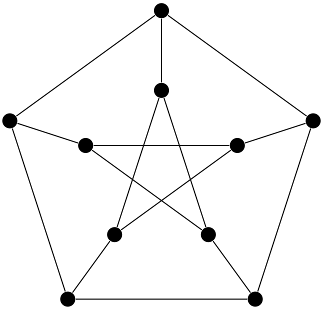
No matter where we put the fifth vertex, it cannot be connected to one of the vertices without crossings.

K_5 is not planar

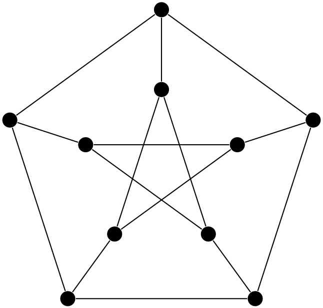


No matter where we put the fifth vertex, it cannot be connected to one of the vertices without crossings.

The Petersen graph



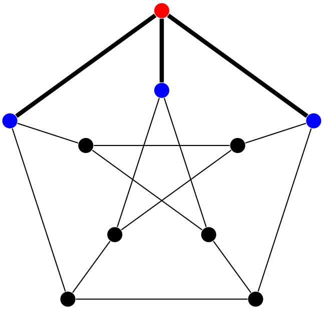
Petersen graph



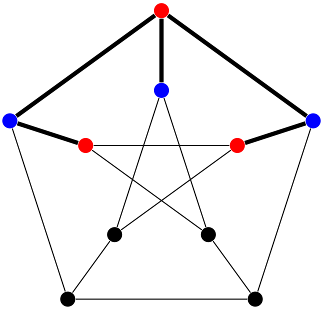
The Petersen graph is not planar, because it contains a subdivision of

Options: K_5; $K_{3,3}$; both K_5 and $K_{3,3}$; don't confuse us, it is planar!

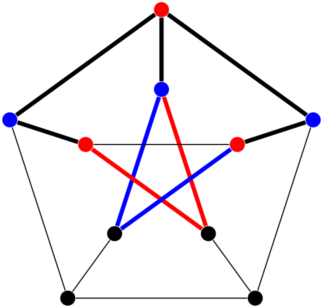
A subdivision of $K_{3,3}$ in the Petersen graph



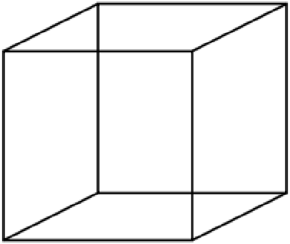
A subdivision of $K_{3,3}$ in the Petersen graph



A subdivision of $K_{3,3}$ in the Petersen graph



Planar graphs and polytopes



Is it true that the graph of the cube is planar?

Planar graphs and polytopes

Is it true that the graph of any convex polytope in $\mathbb{R}^3$ is planar?

Chromatic number and degrees

Which of the following inequalities hold for any graph with maximum degree Δ and chromatic number χ?

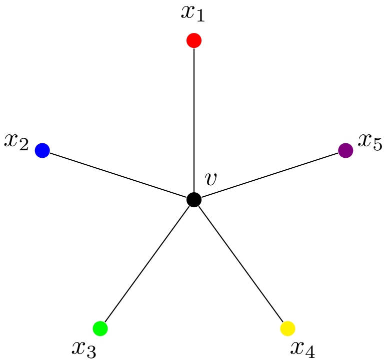
Options: $\chi \leqslant \Delta$, $\chi \leqslant \Delta + 1$, $\chi \geqslant \Delta$, $\chi \geqslant \Delta + 1$.

Chromatic number and degrees

Which of the following inequalities hold for any graph with minimum degree δ and chromatic number χ?

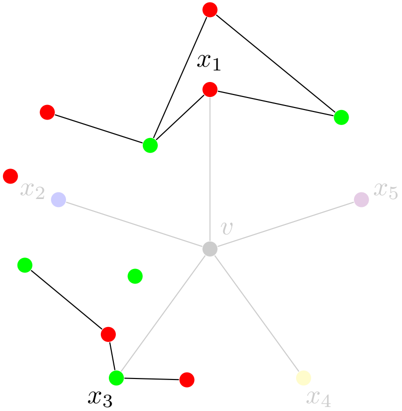
Options: $\chi \leqslant \delta$, $\chi \leqslant \delta + 1$, $\chi \geqslant \delta$, $\chi \geqslant \delta + 1$.

Five-color theorem



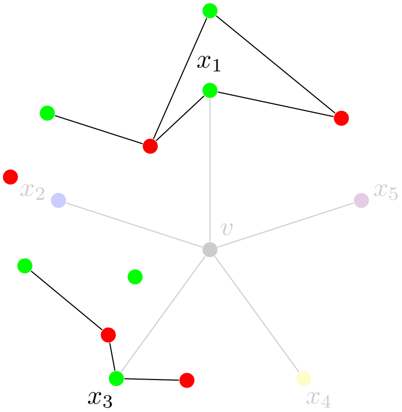
Take v of degree 5 and consider its neighborhood. It must be 5-colored.

Five-color theorem



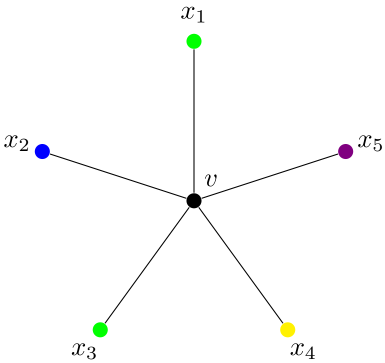
Restrict the attention only to the red and green vertices.

Five-color theorem



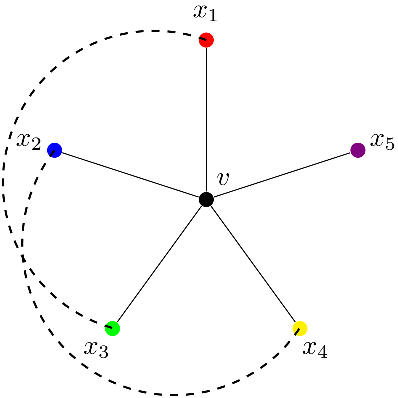
If x_1 and x_3 are not connected in this restricted graph of red and yellow vertices, then switch red and green in the component of x_1. This is a proper coloring.

Five-color theorem



Color v in red.

Five-color theorem



There must be a red-green path between x_1 and x_3. Similarly, there must be a blue-yellow path between x_2 and x_4. They cannot intersect, but they should.