Problem Set 11

Graph Theory 2016 – EPFL – Frank de Zeeuw & Claudiu Valculescu

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You can hand in one of the star problems before 10:15am on Thursday May 26th.

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1. Show that if G is $K_{s,t}$-free, and $s \leq t$, then $|E(G)| \leq \frac{1}{2}(t-1)^{1/s}|V(G)|^{2-\frac{1}{s}} + \frac{1}{2}s|V(G)|$.

2. (a) Let P be a set of n points in $\mathbb{R}^2$ and C a set of n circles. Show that the number of pairs $(p, c) \in P \times C$ such that $p \in c$ is at most $Cn^{5/3}$ for some constant C.
   (b) Let P be a set of n points in $\mathbb{R}^2$. Show that the number of pairs of points from P that have distance 1 is at most $Cn^{3/2}$ for some constant C.

3. Let P be a set of n points in $\mathbb{R}^2$, such that no two points are more than distance 1 apart. Show that there are at most $n^2/3$ pairs of points whose distance is greater than $1/\sqrt{2}$.

4. Let H be a graph. A graph G is called *H-saturated* if G contains no H, but adding any edge to G creates an H. We know that the maximum number of edges in a K_3-saturated graph is $\frac{1}{4}|V(G)|^2$. Determine the *minimum* number of edges in a K_3-saturated graph.

5. Recall that P_k is the path with k edges. Show that if a graph G contains no P_k, then $|E(G)| \leq \frac{1}{2}(k-1)|V(G)|$. (Note that this improves by a factor $\frac{1}{2}$ the bound that we have seen for trees.)

*6. What is the maximum number of edges in a graph on n vertices that has *precisely one* triangle?

*7. What is the maximum number of edges in a C_5-free graph on n vertices?
