

Homework 9 – PROBABILISTIC METHODS

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21 April

Questions

1. Let X be a number chosen uniformly at random from $\{1, \dots, n\}$. Compute $\text{Var}[X]$. What if X is chosen from $\{-k, -k + 1, \dots, 0, \dots, k - 1, k\}$?
2. Suppose we roll a die 100 times. Let X be the sum of the numbers that appear over these 100 rolls. What is the best bound you can give for $\text{Pr}(|X - 350| \geq 50)$?
3. Given two independent random variables X and Y , prove that $\text{Var}[X - Y] = \text{Var}[X] + \text{Var}[Y]$.
4. Construct a random variable to show that Markov's inequality is tight, i.e., given an integer k , construct a random variable X such that $\text{Pr}(X \geq k \cdot E[X]) = 1/k$.
5. Similarly, can you give an example of a random variable to show that Chebychev's inequality is tight?

Bonus Problem. Given n red points $R = \{p_1, \dots, p_n\}$ and n blue points $B = \{q_1, \dots, q_n\}$ in the plane, prove that there is a one-to-one pairing of red points to blue points such that the n line segments in the plane (each defined by the two points in a pair) are disjoint.

10 points.