Introduction to Combinatorics Spring, 2011

Homework 12 – Linear Algebra Method

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Questions

1. Every 1-distance set in $\mathbb{R}^d$ has at most $d+1$ elements.
2. Prove Sauer's theorem by induction on n.
3. Prove the following analog of Sauer's Lemma for uniform families. Let n, l, k be natural numbers, $n \geq l \geq k$ and let $\mathcal{F}$ be an l-uniform family of subsets of an n-element set. If $|\mathcal{F}| > \binom{n}{k-1}$ then $\mathcal{F}$ is (n, k)-dense (i.e., there exists a subset Y of size k such that every subset of Z can be gotten from intersecting Y with the sets in $\mathcal{F}$). (Hint: follow the proof given in class for this new setting, where the columns are labelled with $(k-1)$-element subsets only, and argue that the minimal set Y, for which $g(Y) \neq 0$, must still have at least k elements.)
4. Construct a two-distance set in $\mathbb{R}^n$ with $\binom{n}{2}$ points. Recall that a set of points in $\mathbb{R}^n$ is two-distance iff the distance between every pair of points is one of two values.

Bonus Problem. An $n \times n$ table filled with integers has the property that no two columns are identical. Then prove that there exists a row which can be removed so that the property is still true.

10 points.