

## Exercises Week 7

### 1 Understanding the definitions

1. What is the difference between a topological minor and a minor?
2. Does  $K_7$  contain  $K_4$  as a minor? What about as a topological minor?
3. Let  $v_1, \dots, v_6$  be the vertices of  $K_6$  and let  $G$  be the graph obtained from  $K_6$  by removing the edges  $\{v_1v_2, v_1v_3, v_4v_5, v_4v_6\}$ . Does  $G$  have  $K_5$  as a minor? What about as a topological minor?

### 2 Exercises

1. Show that a bipartite planar graph with  $n$  vertices has at most  $2n - 4$  edges.
2. Prove that  $K_{3,3}$  is not planar.
3. Draw a planar graph that is an edge graph of a regular icosahedron. Can you draw it with straight lines?

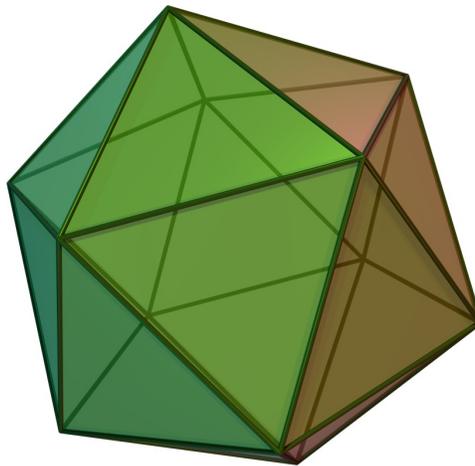


Figure 1: Regular icosahedron.

4. Draw a planar graph that is an edge graph of a regular dodecahedron. Can you draw it with straight lines? Note that the following procedure gives you the planar graph you are looking for. Take the drawing that you did on the previous exercise, draw a vertex in the interior of each face and join the vertices corresponding to adjacent faces. This is called the dual graph, thus dual graphs correspond to dual polyhedrons.

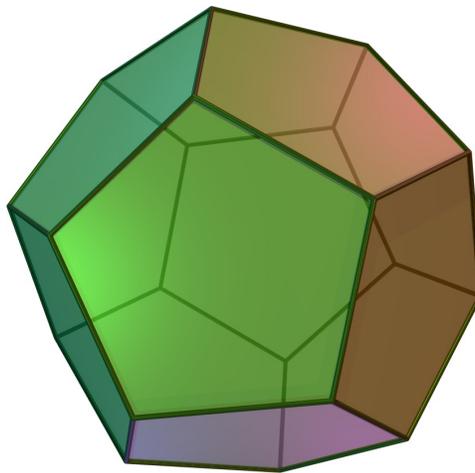


Figure 2: Regular dodecahedron.

5. A platonic solid is a convex polyhedra where each pair of faces are congruent regular polygons where each vertex is incident to the same number of faces. Using Euler's formula, characterize the platonic solids.
6. Let  $G$  be a planar graph with less than 12 vertices. Show that  $G$  has a vertex with degree 4 or less.

**Bonus Problem:** *A football is made of pentagons and hexagons, not necessarily of regular shape. They are sewn together so that their seams form a 3-regular graph. How many pentagons does the football have?*