Prof. Andrei Kupavskii
Assist. Andres Ruiz-Vargas
Claudiu Valculescu

Exercises Week 4

1 Understanding the definitions

For the following practice exercises let $G = (V, E)$ be a graph and A and B be two subsets of V.

1. Assume $|A| = |B| = 2$, what is the maximum possible number of $A-B$ disjoint paths in G.

2. Let X be a subset of V such that X separates A and B. Assume $|A| = |B| = |X| = k$ and that there is a set of k disjoint $A - X$ paths and another set of k disjoint $X - B$ paths. Can you guarantee the existence of k disjoint $A - B$ paths?

3. Let K_n denote the complete graph on n vertices (defined in Assignment 2). Draw the line graph of K_3, K_4 and K_5.

4. Find a graph G such that the line graph of G is K_n.

5. What is the vertex connectivity of K_3 and K_4? What is the edge connectivity of K_3 and K_4?

2 Exercises

1. Find an algorithm that finds a matching of maximum size, for every graph $G = (V, E)$. Your algorithm should run in at most $c|V||E|$ steps, for some (absolute) constant c that does not depend on the size of the graph.

2. Let G be a graph on $2n$ vertices, such that all degrees are at least n. Show that G has a perfect matching.

3. Show that a partially ordered set of at least $rs + 1$ elements contains either a chain of size $r + 1$ or an antichain of size $s + 1$.

Bonus Problem: *Let* $P = \{v_1, ..., v_{n+1}, v_{n+2}\}$ *be a set of* n *points in the plane. For* $i, j > 2$ *we say that* $v_i \prec v_j$ *if the triangle* $v_j v_1 v_2$ *contains* v_i *in its interior. If neither* $v_i \prec v_j$ *nor* $v_j \prec v_i$ *is true, then we say* $v_i \not\sim v_j$. *Show that there exists a subset* $Q = \{y_1, ..., y_{|Q|}\} \subseteq P$ *of size at least* $\sqrt{n}$ *such that either* $p \not\sim q$ *for all* $p, q \in Q$ *or* $y_i \prec y_j$ *for all* $i < j$.