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Exercises Week 10

1 Understanding the definitions

1. Calculate $\text{ex}(n, K_2)$ and $\text{ex}(n, K_n)$.
2. Show that if H is a subgraph of G then $\text{ex}(n, H) \leq \text{ex}(n, G)$.

2 Exercises

1. Let G be a graph and G_1 and G_2 be proper subgraphs of G such that $G = G_1 \cup G_2$ and $|G_1 \cap G_2| \leq 5$. Show that if G has a TK_7 then all of the branch vertices of this TK_7 must be contained either in G_1 or in G_2.
2. Combine the lemmas we have seen in class to prove Kuratowski's theorem (Theorem 4.4.6).
3. Let G be a graph with $\chi(G) = s$. Show that $\text{ex}(n, G) \geq \text{ex}(n, K_s)$.
4. Show that the graph $T^2(n)$ has the maximum number of edges among all complete bipartite graphs on n vertices.
5. Show that the graph $T^r(n)$ has the maximum number of edges among all complete r-partite graphs on n vertices.
6. For a reminder of some basic probability:
   a. Imagine we throw a coin 100 times, what is the expected number of tails?
   b. Imagine we throw a fair dice 5 times. Let X be the sum of the 5 obtained values. What is bigger $P(X \leq 5)$ or $P(X \geq 30)$?
   c. Imagine we throw a fair dice 2 times. Let X be the sum of the two obtained values. Calculate $P(X \text{ is odd})$.
   d. A normal deck consists of 52 cards, 13 of each suit, 2 red suits and 2 black suits. What is the probability of taking two cards at random and obtaining a pair? And what is the probability of taking two cards at random and having a pair of cards with the same colour?

Bonus Problem: *For every $s \in N$ there exists a constant c such that every graph on n vertices with no $K_{s,2}$ graph has at most $cn^{3/2}$ edges.*