ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Prof. Andrei Kupavskii
Assist. Andres Ruiz-Vargas
Claudiu Valculescu

Exercises Week 1: *Introduction*

1. Show that for a graph $G = (V, E)$ we have $\sum_{v \in V} d(v) = 2|E|$.

2. Prove that in every graph the number of vertices of odd degree is even.

3. (a) Is there a graph with degree sequence: $3, 3, 3, 3, 5, 6, 6, 6, 6, 6, 6$?
   (b) Is there a graph with degree sequence: $1, 1, 3, 3, 3, 5$?

4. Let $G = (V, E)$ be a graph and $\delta(G) = \min_{v \in V} d(v)$. Show that G has a path of length $\delta(G)$ and that if $\delta(G) \geq 2$ then G has a cycle of length $\delta(G) + 1$.

   Theorem 1. *The following assertions are equivalent for a graph T:*

   (a) T is a tree;

   (b) Any two vertices of T are linked by a unique path in T;

   (c) T is minimally connected, i.e. T is connected but $T - e$ is disconnected for every edge $e \in T$;

   (d) T is maximally acyclic, e.e. T contains no cycle but $T + xy$ does, for any two non-adjacent vertices $x, y \in T$.

5. Part of the proof of Theorem 1 was shown in class. Show that (c) implies (d) and that (d) implies (a), therefore finishing the proof of the theorem.

6. For a graph $G = (V, E)$ we use $\Delta(G)$ to denote $\max_{v \in V} d(v)$. Let T be a tree. Show that the number of leafs of T is at least $\Delta(T)$.

Bonus Problem: *While at a party with your friends you find yourself on a contemplative mood and you ask yourself, is there two people here (including you) that know the same number of people. Prove that this is indeed the case.*

- The assignment is due on Thursday, Ferbuary 26 at the exercise session
- Submit a solution to the bonus problem *only*

Given out: Thursday, February 19